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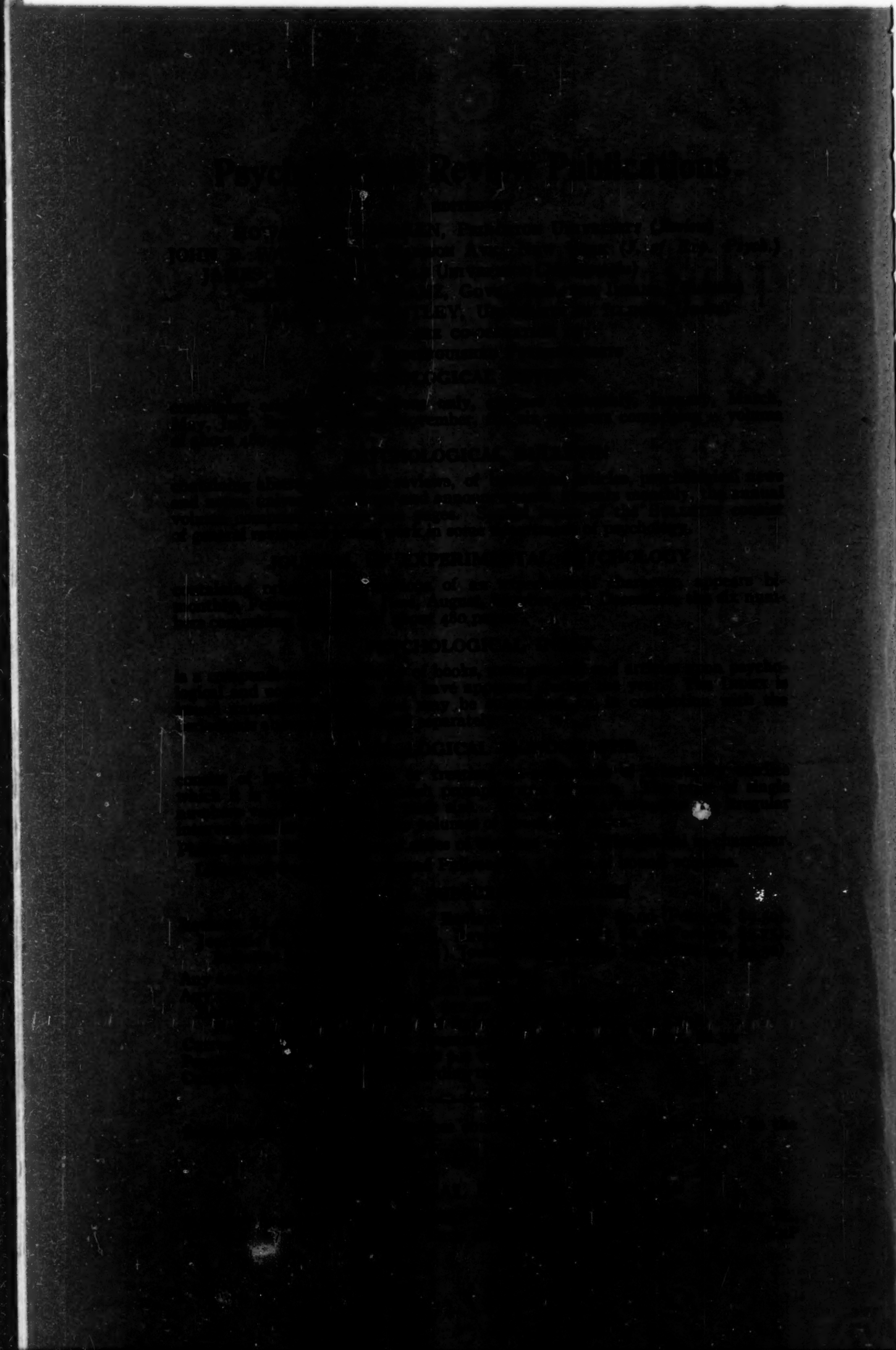

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A MEASURE OF THE ATTENTION

BY H. C. McCOMAS
Princeton University

In the experiment to be described, continuous discrimination time is used as a measure of attention. Inasmuch, however, as attention is only one factor among several, which go into discriminations and reactions, it may seem arbitrary to call the records of such experiments 'measures of attention.' Indeed, much the same sort of method has been used to find variations in efficiency for given tasks, differences in motor impulses and the influence of distractions. Obviously, in recognizing signals, and making movements to indicate such recognition, both receptors and effectors are engaged as well as central processes, and the variations in sense organs and muscles are as inevitably represented in the records as changes in the central processes. The justification for calling continuous discrimination work a measure of attention is simply that the attention is apparently the dominating factor in the time and accuracy of the reactions. This will become evident as the procedure of the experiment is described.

The present attempt to find a 'measure' of attention was actuated by a need in psychiatry for a finer indication of fluctuations than that usually employed, and also the need in applied psychology of some tests of efficiency in tasks which require a vigorous concentration of the attention.

The apparatus used is described in detail in the JOURNAL OF EXPERIMENTAL PSYCHOLOGY, Vol. II., No. 3. Briefly, it consisted in a large cardboard screen with a small ground glass window in the center; behind which were four electric bulbs colored red, green, blue and yellow. When the colored light

from one of the bulbs illuminated the glass the subject, who sat before the screen with two fingers of each hand upon the reacting keys, instantly pressed the key which extinguished that colored light. This reaction not only extinguished that color but immediately presented another, to which the subject again reacted as quickly as possible. This continued for a period of ten minutes. The order in which the different colors appeared was so mixed that it was impossible to anticipate what color would appear. The arrangement of the keys controlling the colors was changed with each experiment, so no habit could be formed between certain keys and certain colors. A record of right reactions and wrong reactions was obtained by a system of markers which recorded upon a kymograph or a paper tape, and the time was indicated in five-second periods. From these records of right and wrong reactions for five-second periods during sittings of ten minutes' duration the material of this paper is prepared. The introspection of each subject was taken.

The apparatus was designed to eliminate the effects of fixation of the eyes and their fatigue, and also the uncertainties which occur in tachistoscopic work when the field is not all exposed instantaneously. With the small movements which were easily and naturally made the effects of fatigue and of learning motor coördinations were reduced to a minimum. If fatigue of either the eyes or the hands entered, it would have been cumulative and the records at the end of a ten-minute test would be characteristically different from the beginning, but this did not appear in the records. On the other hand, the time of reaction was conspicuously lengthened by anything which diverted the attention from the task.

A necessary feature of attention measurement is, of course, the interest the task arouses. If one becomes bored and indifferent obviously the record is vitiated. In these reactions to the colored lights there is an element of amusement. The game of 'putting out the lights' as quickly as possible 'with no wrong moves' proved entertaining to children and to elderly people.

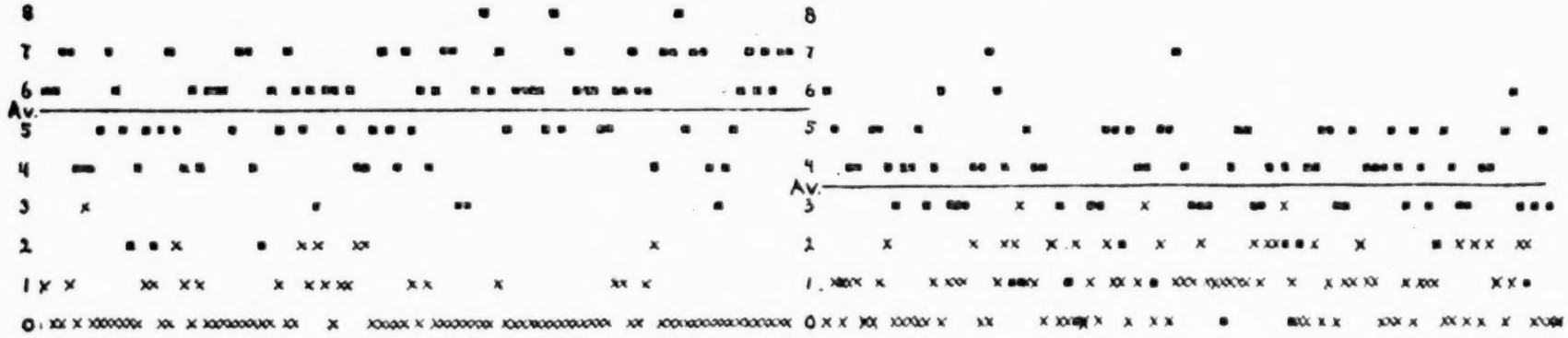
Eleven subjects took part in the work, which was conducted during the second terms of two college years. In the first year *A* and *B* were graduate students with two years' laboratory experience, *C* was an instructor and *D* a professor in psychology. *D* acted as subject in both years; his first year's work is indicated by D^1 and the second year by D^2. In the second year *E* was an instructor and *F* and *G* graduate students of one and three years' laboratory experience respectively. *H* was a graduate student who was also teaching in the high school; this was his first year in laboratory work. *I* was a girl of fifteen, *J* and *K* were boys of fifteen from the high school.

Records were made during the first year upon a kymograph with the Jaquet marker registering time in one-second periods. During the second year a Foot-Pierson telegraph recorder was substituted for the kymograph and a pendulum with electric contacts was used, in place of the Jaquet. It also registered in one-second periods. In reading the records the right and wrong reactions were first grouped into five-second periods for analysis. As it is impractical to give the actual records, two specimens for eight minutes' work are given in Chart I. These were selected from the work of subject *G* who showed a greater variation in his reactions than any other subject.

Four items are of importance in every record: the averages for the right and the wrong reactions and the variation from these averages. Chart I. illustrates these features. The average of right reactions per five-second period on the eighth day is 5.5 as compared with 3.7 for the fourth, while the average for wrong reactions has risen on the fourth day to .85 from that of .33 for the eighth day. Moreover the variations from the averages has noticeably increased in the records of the fourth day.

The relation of right reactions to wrong is very important. If a subject gives no wrong reactions and his right reactions are comparatively few it is impossible to tell whether he is reacting as quickly as he can or not, for it is obvious that his limit may be above that he is achieving. On the other hand

CHART I. Discrimination reactions, 5-sec. periods.



8th Day
Horizontal Direction = Successive Reactions.

4th Day
Vertical Direction = Number of Reactions.

a large number of false reactions show that the subject is taking chances on his reactions.

Indeed the record for *G* on the eighth day of his experimental work is one of the best he made. The average number of reactions per five-second period is very high for him and the variations from the average are not very large. His wrong reactions are few. But in his record on the fourth day his averages tell another story. He is not as quick, nor as accurate, and there are several five-second periods in which he made no reactions at all.

This last feature is so exceptional that some explanation is necessary. It seems incredible that a subject should remain before a signal for ten seconds, on two occasions in a ten-minute sitting, and register nothing but a few false reactions. Ordinarily such a record would be thrown out as indicating a lack of coöperation on the part of the subject. In the present instance the subject was thoroughly interested in the experiments, volunteered willingly to react, made many sacrifices to be present regularly and gave every assurance of endeavoring to coöperate to the best of his ability. As a matter of fact he was not aware that he made such extensive lapses as his records show, though they appeared quite frequently. In an experiment on continuous simple reactions he showed the same tendencies. His strong tendency is to attend for several minutes and then relax his effort. (In writing an article he would habitually lay down his pen and look out of the window for a few moments and then go back to his task.) According to his introspection these lapses in his record are due to his attention wandering from the task. Sometimes the apparatus would start a train of ideas, which diverted him; occasionally worry about his private affairs intruded and for several seconds his attention be away from the colored light and upon some irrelevant matter. So often did this occur when he was obviously trying to correct the fault and to give a good record that it can only be attributed to a habit of relaxing his effort, or a native trait in the fluctuation of his attention.

Other subjects gave some indication of this trait, but none to the same degree. How typical this is cannot be stated from a small group of eleven subjects. Nevertheless, when it is recalled that eight of these subjects were highly educated men of more than average intelligence it would seem at least probable that such lapses are even more typical of men of less mental discipline. Several workers in applied psychology find this trait a possible explanation for lapses in attention among chauffeurs and engineers which result in disasters otherwise inexplicable.

Chart I. shows the data when the right and wrong reactions are grouped in the five-second periods. Such a grouping is evidently too restricted; for the scattering of the data obliterates any large trends which might be evident. It was therefore found advisable to group the right and wrong reactions in thirty-second periods. Such a grouping smoothes out the evident eccentricities of the data and permits an analysis of trends. The material in the tables is given on the basis of the thirty-second period.

A survey of the tables shows that there is frequently an increase in the number of right reactions when there is an increase in the wrong. The subject is trying to obtain his greatest speed possible with accuracy and frequently outspeeds his accuracy.

Naturally some relation exists between these phenomena. It is the relation which constantly presents itself in experiments involving speed and accuracy. If there were a high correlation between the right and wrong reactions it would be possible to use the 'line of regression' as a means for determining what gain in speed is obtained by the increase in inaccuracy; but there is no such guide; this is because, in addition to the rise in the number of right reactions due to the subjects becoming less accurate, there is also a rise due to a learning curve. It is necessary, however, to compare the performances of the subjects from time to time and, in order to do this, there must be some one expression which represents the number of correct reactions.

To penalize the record of right reactions by subtracting one or two for each wrong reaction would give a score in which both the speed and the accuracy of the reactions are represented, but it would be purely arbitrary. In view of the fact that the wrong reactions are the result of too great haste; or lack of attention, so that the process of recognizing the signal and selecting the correct response is disturbed, and the question of which key is pressed is a matter of chance; we may assume that the increase of the wrong reactions is an indication that the subject is relying less upon careful discrimination and choice of reactions and more upon chance. Chance, then, becomes the prominent factor in the situation. Now, if the subject reacted as quickly as his fingers could move and quite regardless of the colors of the stimuli he would hit the right key once in four trials. In pure chance reacting he would be right with 25 per cent. of the reactions and wrong with 75 per cent. If 75 per cent. of the reactions due to chance are wrong we may assume that in each thirty-second period the wrong reactions constitute 75 per cent. of the chance reactions and that the remaining 25 per cent. of the chance reactions are scattered among the right reactions. If these chance reactions are subtracted from the right reactions we have a figure which adequately represents the subject's reactions not due to chance. Such a figure is serviceable in making a comparison of the different subjects at different times and for plotting curves upon a chart. It is readily obtained by subtracting one third of the number of wrong reactions from the total number of right reactions. By this formula the records of the tables have been entered upon the charts.

Table I. gives the average number of reactions both right and wrong for a series of 10-minute experiments with subjects *A*, *B* and *C*. The recording did not begin until each subject had become thoroughly oriented and when it appeared that the learning curve had levelled out. This, however, was hardly the case with any of the subjects. Some trace of a learning curve, or rather an 'adaptation' curve, appears in

TABLE I

AVERAGE RIGHT AND WRONG REACTIONS IN HALF-MINUTE PERIODS

Experiment	Subjects											
	A				*B*				*C*			
	Right	M. V.	Wr'g	M. V.	Right	M. V.	Wr'g	M. V.	Right	M. V.	Wr'g	M. V.
1	29.1	1.6	0.3	0.5	29.6	1.8	0.8	0.7	38.8	1.7	6.4	2.1
2	27.2	2.7	1.3	1.1	30.0	1.7	1.3	1.0	35.2	1.8	6.7	2.8
3	32.8	1.8	1.1	1.1	28.2	2.1	1.3	0.8	37.4	1.8	7.5	2.5
4	28.8	1.7	1.4	1.1	31.6	2.0	1.9	1.6	37.1	1.7	10.1	1.7
5	31.4	1.8	0.6	0.7	30.1	2.7	1.7	1.0	37.9	1.4	8.3	2.0
6	26.8	2.6	4.3	4.3	30.3	2.6	2.1	1.0	36.2	1.9	6.7	1.9
7	30.1	1.9	1.3	0.9	31.6	2.4	2.5	0.9	37.7	2.5	7.0	2.4
8	26.7	5.6	0.4	0.6	37.7	1.9	3.1	1.3	41.6	2.0	5.6	2.6
9	28.0	2.0	1.5	1.2	33.0	2.5	3.6	1.6	37.8	2.0	5.5	2.1
10	29.3	3.4	4.6	1.3	...	...	...	...	...	...	...	...
11	29.2	2.2	0.3	0.4	...	...	...	...	...	...	...	...
12	31.6	2.7	1.0	0.9	...	...	...	...	...	...	...	...
13	32.2	2.8	1.9	1.0	...	...	...	...	...	...	...	...

the table. The experiments usually occurred once each week, but in many cases lapses of one or more weeks were inevitable. Despite the irregularity in the averages from day to day and their tendency to rise, it is clear that there are distinct differences in the subjects. *C* is quicker, but less accurate than *A* and *B*. He does not, however, scatter his reactions as widely as they, as is shown by his smaller mean variations. *B* gradually increased his errors in his effort to better his records.

These records for continuous discrimination reactions show that the average time of reaction for each subject is longer than the usual discrimination time. Thus, the averages ran as follows: *A* = .93 sec. (m.v., .08), *B* = .90 sec. (m.v., .05) and *C* = .77 (m.v., .04). The reason for this lengthened time is obvious. The usual method of obtaining discrimination reactions permits the subject a period of preparation for each reaction before the stimulus is presented, often a signal of warning is given. This results in a preparation which brings the stimulus at the moment of maximum alertness. Moreover, the subject can rest between reactions. The discrimination time may not be taken until the subject has formed

habits of reacting in the same way to the same stimulus, which, of course, is obviated in the present method. For these reasons the time of reaction for all of the subjects is not comparable to that obtained in the ordinary discrimination work.

TABLE II

Average Number of Reactions per Half Minute for Subject *D* (1)					Average Number of Reactions per Half Minute for Subject *D* (2)				
	Right	M.V.	Wrong	M.V.		Right	M.V.	Wrong	M.V.
1st Day					*1st Day*				
7.30 A.M. . .	33.8	3.0	4.7	1.6	8.30 A.M. . .	27.4	3.4	3.3	1.2
12.30 P.M. . .	28.6	1.6	5.8	1.6	12.30 P.M. . .	26.4	2.2	3.9	1.7
6.00 P.M. . .	27.4	1.9	4.6	2.0	——— . .	—	—	—	—
10.15 P.M. . .	30.6	2.2	6.1	1.8	10.30 P.M. . .	30.1	1.6	3.5	1.2
2d Day					*2d Day*				
7.30 A.M. . .	33.4	2.2	3.4	1.0	8.30 A.M. . .	29.9	2.0	2.8	1.0
12.30 P.M. . .	31.7	3.0	5.0	1.7	12.30 P.M. . .	25.7	2.9	3.2	1.6
6.00 P.M. . .	30.7	2.6	3.9	1.5	1.30 P.M. . .	23.2	2.8	4.1	2.6
10.15 P.M. . .	34.0	1.3	3.6	1.1	10.30 P.M. . .	—	—	—	—
3d Day					*3d Day*				
7.30 A.M. . .	33.8	2.1	3.2	1.2	8.30 A.M. . .	32.0	1.0	2.5	1.7
12.30 P.M. . .	28.6	2.2	5.0	1.4	1.00 P.M. . .	27.5	2.4	3.5	1.2
6.00 P.M. . .	32.6	2.6	4.2	1.7	6.00 P.M. . .	28.0	2.5	1.5	1.0
10.15 P.M. . .	27.3	1.7	5.2	1.5	10.30 P.M. . .	28.3	2.2	3.1	1.2
4th Day					*4th Day*				
7.30 A.M. . .	32.9	2.7	2.2	0.9	8.30 A.M. . .	29.6	3.3	2.1	0.9
12.30 P.M. . .	30.0	2.0	3.8	1.2	1.00 P.M. . .	30.0	1.6	3.0	0.6
6.00 P.M. . .	31.3	2.0	3.7	1.8	6.00 P.M. . .	26.3	2.2	4.0	0.3
10.15 P.M. . .	32.7	2.4	3.4	0.9	10.30 P.M. . .	25.0	2.3	6.8	1.6
5th Day					*5th Day*				
7.30 A.M. . .	35.9	2.5	2.8	0.9	8.30 A.M. . .	26.6	3.0	4.0	1.2
12.30 P.M. . .	32.5	2.3	3.9	1.2	1.00 P.M. . .	27.7	2.7	4.3	1.5
6.00 P.M. . .	31.5	3.0	4.8	1.5	6.00 P.M. . .	32.0	1.2	3.5	1.4
10.15 P.M. . .	32.5	2.0	4.3	1.3	10.30 P.M. . .	27.8	2.3	4.0	1.3
6th Day					*6th Day*				
7.30 A.M. . .	36.6	3.1	4.2	1.3	8.30 A.M. . .	32.5	1.8	2.7	1.6
12.30 P.M. . .	34.8	2.7	4.8	1.3	12.30 P.M. . .	31.1	2.4	3.5	1.6
6.00 P.M. . .	32.4	3.6	5.8	2.0	1.30 P.M. . .	31.5	3.0	3.9	1.0
10.15 P.M. . .	35.3	3.1	4.9	1.5	10.30 P.M. . .	31.1	2.1	4.0	1.4

Table II. gives the records for subject *D* in the first and second years of the experiment. During the first year's work he carried a heavy schedule in teaching and laboratory work. The second year was less laborious and the experiments were made at approximately 8:30 A.M. instead of 7:30 A.M. In

both years the second experiment of the day occurred about 12:30 P.M. and the third about 6 P.M., though variations of a half hour from the prescribed time were frequent. The last experiment, at 10 P.M., was made after the close of two hours in a seminar.

The averages for the number of right reactions per thirty-second periods are lower in the second year than in the first until the sixth day. Possibly the explanation lies in the fact that during the second year the subject was constantly worried by a case of illness in his family and did not give the same undivided attention to his work.

The clearest feature of the first year's reactions is the diurnal curve, which appears when the averages for the four experiments each day are made, after deducting a per cent. for the wrong reactions according to the formula.

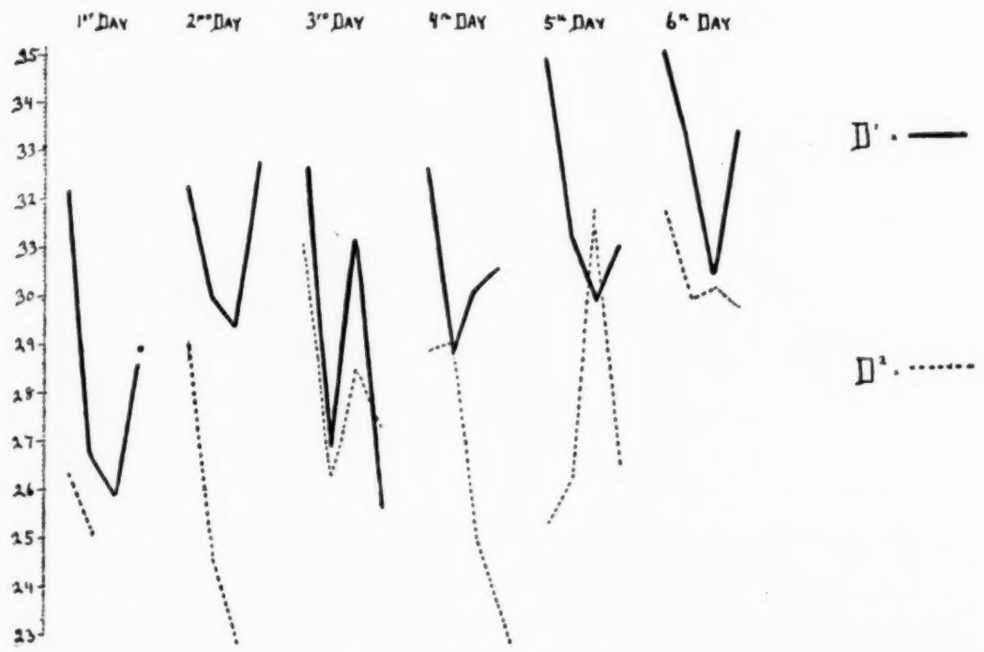


CHART II. Reactions per 30-sec. periods, taken 8 A.M., 12:30 P.M., 6 P.M., 10:30 P.M.

Chart II. shows the diurnal curve for *D*'s first year's work in the solid lines. It will be seen that in every instance, during this year's work, the early morning results were the best with one exception, and in every intance but one the results at night were superior to those in the late afternoon. These curves were

so suggestive of a diurnal variation that the experiment was carried on the second year with a view to determining whether the curves were fortuitous or not. The dotted line in the chart shows the averages for the second year. The records for the first and second day are not complete, the third experiment for the first day and the fourth for the last are missing. These dotted curves show no evidence of a diurnal variation. They agree with the findings of the first year in that four out of the six days show better averages in the early morning work than during the day, and this would seem to be the only safe inference from the two sets of curves.

As a further test of diurnal possibilities three graduate students *E*, *F* and *G*, went through the experiments in the morning, midday, afternoon and night. Their results appear in Table III. Several experiments are lacking, to complete the four for each day, with subjects *E* and *F*. Nevertheless it is very evident that there is no semblance of a diurnal curve among these subjects. There is however a great difference in the efficiency shown on many days when we use the formula mentioned for comparisons. Thus, there is a drop for *E* on the third day between morning and midday of 15 per cent., of which the subject was not aware. His introspection is to the effect that his 'attention wavered' more in the morning, and that the work at midday was better. His improvement in efficiency of reaction from late afternoon to night is 27 per cent. The next day his first three records are slightly better in right reactions than any preceding day, while the night record shows a drop of about 11 per cent.; and, again, the introspection contradicts the records, for the subject thought he did his best work at night.

The greatest change subject *F* shows in any one day's records occurs in the third day when his record shows a gain of about 19 per cent. in efficiency. His introspection shows he was aware of this improvement. He was overworked, tired and sleepy all day, he felt somewhat better in the morning, when he was doing the poorer work.

Subject *G* was convinced that his best work, that which required sustained attention, was always done late at night.

TABLE III

Average Number of Reactions per Half Minute for Subject *E*					Average Number of Reactions per Half Minute for Subject *F*					Average Number of Reactions per Half Minute for Subject *G*				
	Right	M.V.	Wrong	M.V.		Right	M.V.	Wrong	M.V.		Right	M.V.	Wrong	M.V.
1st Day					*1st Day*					*1st Day*				
9.00 A.M.	33.2	1.5	3.2	2.1	9.00 A.M.	26.4	2.1	1.5	1.3	10.00 A.M.	25.1	3.1	1.8	1.0
12.30 P.M.	36.0	1.7	3.3	1.0	1.00 P.M.	25.3	2.0	1.1	1.9	1.00 P.M.	27.0	2.0	1.2	0.5
6.00 P.M.	34.0	3.4	3.4	1.2	6.00 P.M.	26.5	2.8	2.6	1.1	6.00 P.M.	26.0	2.0	1.3	0.9
9.30 P.M.	33.2	3.4	1.5	1.3	10.00 P.M.	26.8	2.7	1.6	1.3	10.30 P.M.	26.2	2.3	1.3	0.7
2d Day					*2d Day*					*2d Day*				
9.00 A.M.	36.6	2.3	4.4	1.5	9.00 A.M.	30.2	2.2	0.5	0.6	10.00 A.M.	27.2	4.0	0.8	0.3
12.30 P.M.	39.5	1.7	3.2	1.5	1.00 P.M.	31.0	2.0	3.3	2.0	1.00 P.M.	21.0	4.4	3.5	2.1
6.00 P.M.	34.2	3.3	4.5	1.6	6.00 P.M.	32.4	2.4	2.3	1.2	6.00 P.M.	20.8	5.1	3.7	2.0
9.30 P.M.	35.2	2.0	4.9	1.5	10.00 P.M.	31.4	1.6	0.1	0.3	10.50 P.M.	26.6	3.0	0.2	0.3
3d Day					*3d Day*					*3d Day*				
9.00 A.M.	36.2	1.6	4.2	1.6	9.00 A.M.	28.2	2.1	5.2	2.9	10.00 A.M.	26.9	3.4	0.5	0.5
12.30 P.M.	31.6	3.1	6.2	1.2	1.00 P.M.	28.9	3.2	7.4	2.4	1.00 P.M.	24.4	2.9	0.2	0.4
4.00 P.M.	32.5	3.2	6.6	2.1	6.00 P.M.	29.7	2.9	3.9	1.3	6.00 P.M.	24.7	3.6	0.7	0.6
9.00 P.M.	41.4	1.4	0.6	0.4	10.00 P.M.	36.7	2.0	5.4	1.4	10.30 P.M.	24.1	2.0	2.3	1.4
4th Day					*4th Day*					*4th Day*				
9.00 A.M.	43.0	2.6	4.1	1.6	9.00 A.M.	33.8	2.7	7.1	1.9	10.00 A.M.	26.9	3.6	3.2	1.7
12.30 P.M.	42.0	1.9	3.7	1.0	1.00 P.M.	32.2	2.4	6.5	1.5	1.00 P.M.	23.0	4.4	6.1	2.4
5.30 P.M.	42.6	2.0	5.9	1.9	6.00 P.M.	29.7	3.0	4.0	2.0	6.00 P.M.	23.1	3.4	7.0	2.4
10.00 P.M.	39.0	2.4	8.7	2.0	10.00 P.M.	...	...	...	...	10.30 P.M.	20.6	3.2	5.8	2.2

TABLE III *Continued*

Average Number of Reactions per Half Minute for Subject *E*					Average Number of Reactions per Half Minute for Subject *F*					Average Number of Reactions per Half Minute for Subject *G*				
	Right	M.V.	Wrong	M.V.		Right	M.V.	Wrong	M.V.		Right	M.V.	Wrong	M.V.
5th Day					*5th Day*					*5th Day*				
9.00 A.M. . . .	48.9	3.0	10.0	5.5	9.00 A.M.	36.9	1.0	6.8	1.5	10.00 A.M.	20.8	5.4	4.7	1.4
. . . . .	. . .	. . .	. . .	. . .	1.00 P.M.	35.5	1.6	5.8	2.2	1.00 P.M.	24.9	2.7	2.9	1.3
6.00 P.M. . . .	45.9	2.5	9.9	3.1	6.00 P.M.	34.8	3.1	6.8	1.5	6.00 P.M.	28.0	3.5	2.0	0.4
10.00 P.M. . . .	46.6	2.3	9.7	1.7	10.00 P.M.	34.8	1.5	5.3	1.6	10.30 P.M.	28.0	5.5	2.1	1.1
6th Day										*6th Day*				
9.00 A.M. . . .	41.8	2.0	12.8	1.8						10.00 A.M.	24.0	5.0	1.7	1.2
1.00 P.M. . . .	42.6	2.7	12.2	3.0						1.00 P.M.	28.7	3.0	5.0	1.2
6.00 P.M. . . .	43.7	2.3	10.3	1.9						6.00 P.M.	23.1	4.3	4.9	1.8
9.00 P.M. . . .	43.4	2.1	9.4	2.6						10.30 P.M.	25.3	4.2	3.9	2.0
										7th Day				
										10.00 A.M.	21.1	3.3	2.2	1.7
										1.00 P.M.	26.7	3.2	2.5	1.7
										6.00 P.M.	30.2	3.1	3.1	1.3
										10.30 P.M.	28.1	4.6	1.9	1.6
										8th Day				
										10.00 A.M.	29.1	4.3	4.3	1.3
										1.00 P.M.	30.0	2.4	2.9	2.0
										6.00 P.M.	33.3	2.7	2.5	2.3
										10.30 P.M.	30.7	4.1	5.9	2.1

Habits of years' standing confirmed him in this conviction. His records for attention in this type of work, however, do not show any superiority of the night reactions over those taken in the morning. Often *G* did better work when he felt fatigued, for then his mind seemed less alert and fewer distracting ideas occurred to him.

In general the introspection of these subjects was unable to detect variation in efficiency of 10 per cent., occasionally more, if by 'efficiency' is meant success in performing the task of reacting quickly and accurately. This suggests the inadvisability of relying on one's own judgment as to the character of work done under similar conditions, such as speed and accuracy in taking and sending telegraph messages, in doing linotype work, sorting type, court stenography and such tasks which are performed under pressure for quick and correct results.

Though it is evident from the four daily records of *E*, *F* and *G* that there is no diurnal curve, the results of *H* show the very clear effect of a day's work in teaching upon efficiency in discrimination reactions. See Table IV. In the seven

TABLE IV

AVERAGE NUMBER OF REACTIONS PER HALF MINUTE FOR SUBJECT *H*

	Right	M.V.	Wrong	M.V.
1st Day				
8.30 A.M.	38.0	1.4	2.5	0.7
5.00 P.M.	35.6	1.5	3.0	1.3
2d Day				
8.30 A.M.	38.0	1.7	4.3	1.6
5.00 P.M.	37.7	1.7	3.3	1.0
3d Day				
8.30 A.M.	42.3	1.6	3.0	1.6
5.00 P.M.	40.2	1.6	5.5	1.3
4th Day				
8.30 A.M.	42.1	1.6	5.1	1.5
5.00 P.M.	41.1	2.3	5.6	1.2
5th Day				
8.30 A.M.	44.1	2.4	10.0	2.9
5.00 P.M.	41.3	2.4	10.2	2.4
6th Day				
8.30 A.M.	46.2	1.5	8.7	1.8
5.00 P.M.	41.3	2.4	11.1	1.8
7th Day				
8.30 A.M.	46.2	1.9	11.4	1.6
5.00 P.M.	47.4	2.0	12.1	2.8

days' experiments there was a drop in the number of right reactions each afternoon in six instances and an increase in wrong reaction in six instances. Obviously the school work, with subject *H*, was sufficiently fatiguing to affect his ability to concentrate upon his reactions as well in the afternoon as in the morning. His introspection, however, did not discover this in all cases. On the fifth and sixth days he was aware of the fact that he was not making as good records in the afternoon as in the morning, but he felt that he was more accurate in his reactions on the afternoon of the fourth day than in the morning. A comparison of the day's work (Chart

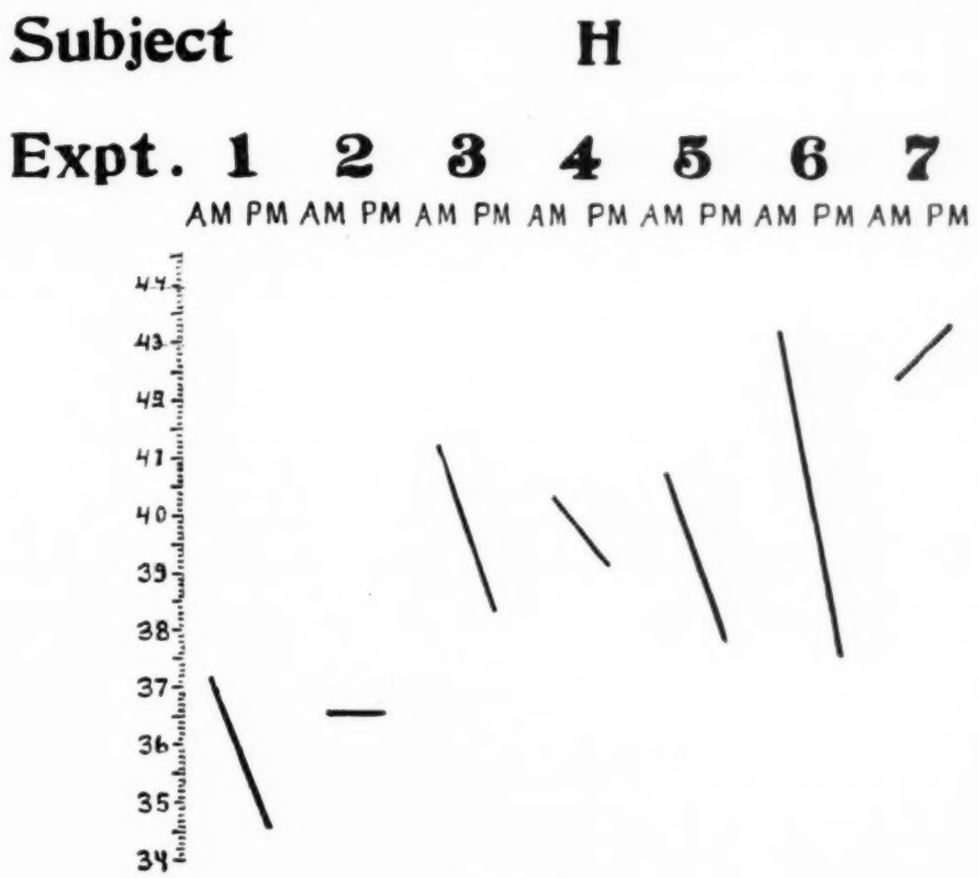


CHART III

III) will show the differences in the efficiency in the morning and afternoon work when the deduction for chance reactions is made according to the formula previously given. These differences are the more impressive as there is a learning curve apparent in the series. Nevertheless, this learning

factor did not make the afternoon records superior to those of the morning, a condition which appears in the results of subjects *I*, *J* and *K*. It should be stated that *G*'s school work involved seven periods of teaching, and also a period of correcting papers.

It was not practicable to run the subjects *I*, *J* and *K* through a long series of tests. The records were taken after each of the subjects had become familiar with the apparatus and the requirements of the test, and had made sufficient number of reactions to indicate the large rise in the learning curve had been passed. Table V. shows that there was still a learning factor in their results.

Table V

Average Number of Reactions per Half Minute for Subjects *I*, *J*, *K*

Subject I

	Right	M.V.	Wrong	M.V.
1st Day				
1 P.M.	24.9	2.6	5.5	2.2
5 P.M.	25.2	3.4	4.2	1.5
2d Day				
1 P.M.	31.2	2.7	4.8	1.4
5 P.M.	26.9	3.2	9.7	3.1

Subject J

1st Day				
1 P.M.	27.2	1.8	0.8	0.6
5 P.M.	29.7	3.0	1.3	1.0
2d Day				
1 P.M.	29.2	2.9	2.3	1.1
5 P.M.	31.7	2.3	1.7	0.7
3d Day				
1 P.M.	27.4	3.5	2.6	0.9
5 P.M.	26.3	4.3	2.8	1.5

Subject K

1st Day				
1 P.M.	34.1	2.0	4.3	1.8
5 P.M.	36.8	2.0	3.2	1.7
2d Day				
1 P.M.	36.5	2.6	4.0	1.8
5 P.M.	35.9	2.0	3.9	1.8
3d Day				
1 P.M.	38.2	1.3	5.0	2.2
5 P.M.	36.9	2.4	6.5	2.1

The tests were made to determine whether an hour and a half of close application to memory work would affect the attention for the type of task represented by the tests. The memory work involved a rigorous concentration of the attention, which was sustained for the entire period of the experiment. Immediately after this the subjects took the discrimination reaction tests. A comparison of the work done at 1 P.M. and at 5 P.M. on the last day of the testing shows the effect of the memory work upon the reaction tests. The first day with subject *I*, and the first two days with subjects *J* and *K* involved no memory work, and in every case but one there is an improvement in the late afternoon work over that done earlier in the afternoon. After this the subjects took the discrimination reaction tests. Subject *I* shows some improvement in her reactions both for time and accuracy, in the records for her tests at five o'clock, as compared with those for one o'clock, on the first day. The second day her records for the one o'clock work were very much superior to those of the preceding tests, but after the hour and half memory work her quickness of reaction had dropped and her errors had risen conspicuously.

Subject *J* showed a marked improvement in his late afternoon work on both of the days preceding the memory work tests, but after the memory work on his third day he is not so quick and accurate as before; indeed, he gives his poorest record then.

Subject *K* was a quicker reactor than *I* or *J*. His records on the first day show an obvious improvement in the late afternoon work, but the records for the second day do not show this, though the record for the late afternoon work is but slightly below that of the earlier work. However, on the third day his record taken after the memory work shows a clear deterioration in both speed and accuracy of reactions.

It would appear from the results of *H*, *I*, *J* and *K*, that any work which calls for close application during a portion of the day will affect the ability to concentrate the attention,

and the results of *E*, *F* and *G* indicate that this effect is more conspicuous than a diurnal variation. *D*'s results corroborate such an assumption.

Conclusions

While no broad generalizations can be drawn from the results of eleven subjects, it would appear that under the conditions of the experiment the following conclusions may be drawn:

1. Continuous discrimination reactions may be used as a means for detecting variations in attention.

2. The time obtained for such continuous discrimination reactions averages longer than that obtained when an appreciable time interval intervenes between a reaction and a succeeding stimulus.

3. Introspection gives a very inadequate description of variations in efficiency in this type of work.

4. Such diurnal variations as appear are due to effects from specific tasks.

5. Conspicuous differences in speed and accuracy appear even in a small and highly selected group of subjects.

EFFECTS OF LOSS OF SLEEP (I.)

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Introduction

The experiments reported in this paper and in the one to follow it [1] were conducted with the purpose of studying the effects of loss of sleep. We were interested in the specific nature of these effects as well as in their general amount and direction.

The qualitative effects of insomnia are apparently quite definite. Roemer[2] and Aschaffenburg[3] have both reported that exhaustion brought on by loss of sleep characteristically increases the proportion of sound and other lower grade associations in the free association test without materially increasing association time. Patrick and Gilbert [4] have described visual hallucinations and other incidents of dissociation and disorganization. Visual hallucinations were also noted by Smith,[5] and Aschaffenburg[6] observed occasional hallucinations of contact during one of his experiments. Hypersensitivity and emotional instability were noted by Smith.

From a quantitative standpoint the outstanding paper is the one already cited, by Smith. She is the first investigator of this subject who, so far as we are aware, has obtained any-

[1] The second study was carried on by E. S. R. and F. Richardson-Robinson.

[2] Roemer, E., 'Ueber einige Bezielungen zwischen Schlaf und geistigen Taetigkeiten,' Dritter internationaler Kongress für Psychol., 1896, 353–355 (a brief summary).

[3] Aschaffenburg, G., 'Experimentelle Studien über Associationen,' *Psychol. Arbeiten*, 1895, I., 209–299, and 1897, II., 1–83.

[4] Patrick, G. T. W., and Gilbert, J. A., 'On the Effects of Loss of Sleep,' *Psychol. Rev.*, 1896, 3, 469–483.

[5] Smith, M.—A 'Contribution to the Study of Fatigue,' *Brit. J. of Psychol.*, 1916, 8, 327–350.

[6] Aschaffenburg, G., 'Ueber die psychischen Erscheinungen der Erschoepfung,' *Arch. für Psychiatrie*, 1893, XXV., 594–597.

thing like adequate records from tests preceding and following the period of insomnia. The availability of her report renders it unnecessary to describe her work in any detail. Suffice it to say that in two tests in particular (the McDougall dotting machine and an associated words test) she found that her performance improved or remained normal for a day or so immediately subsequent to a period of insomnia. Following this, there set in a period of inefficiency from which she recovered gradually through some days. An interesting result appears in the fact that during her periods of low efficiency following loss of sleep, her efficiency could be brought back to normal rather abruptly by the interpolation of another period of insomnia. It appears that a mixture of physical weariness and mental exhaltation were present during the early period following loss of sleep when efficiency was relatively high. During the period of lower efficiency, feeling tone was practically normal.

Although Smith's results are consistent throughout her study, it does not seem to us that they can be accepted without further verification. In the first place, she employed but one subject, and, in the second place, her findings are quite unlike those of other investigators. It should be said that she herself admits her experiments might well be carried out in more extensive fashion.

The quantitative results of Aschaffenburg, Roemer, and Patrick and Gilbert show that as a result of periods of enforced wakefulness some functions show a marked deterioration while others show little or no modification. The absence of data from adequate tests before and after the loss of sleep, however, make difficult an estimate of the importance of most of those effects. In the case of Aschaffenburg's and Patrick and Gilbert's experiments a certain complication is present due to the fact that tests were applied continuously or at comparatively short intervals throughout the period of wakefulness. A certain amount of variation in these cases can probably be laid to the specific work of taking the tests rather than to the loss of sleep. But in general it can

be said that the results of these investigations show a loss in efficiency as a result of deprivation of sleep rather than an initial gain followed by a loss.

There is another point of some importance upon which the data of Patrick and Gilbert and those of Smith are not in agreement. The former experimenters found from a single testing, conducted following a 90- (approximately 90-) hour insomnia and the first subsequent night of sleep, that that night of sleep, which was only from 16 per cent. to 35 per cent. longer than an ordinary one, completely restored the efficiency of the subjects employed. They present an explanatory fact in the form of the extreme soundness of the sleep of one of their subjects just after his prolonged wakefulness. While Smith noticed a prompt recovery from *subjective* consequences of loss of sleep, her *objective* results show that the process of recovery was a very gradual one extending over a number of days. Of course, it is possible that further tests upon Patrick and Gilbert's subjects would have shown the apparent restoration only temporary, and that, following it, there was a period of low efficiency lasting for a considerable length of time. There is no tangible evidence, however, for any such assumption.

Because of the disagreements which we have pointed out, it seemed to us desirable to conduct further experiments with the hope of discovering whether the stimulating effects of insomnia reported by Smith are at all general and of uncovering or verifying other consequences of loss of sleep.

Subjects

On account of the arduous nature of our first experiment, only three subjects were employed. K. R. J. is an undergraduate student, male, 20 years of age. While not overweight, he is large and athletic. At the time of the experiment he seemed in excellent health. W. H. is a graduate student, male, 22 years of age. At the time of the experiment, he was in only fair health. S. H. is a graduate student, male, 22 years of age. His health was very good.

Method

In our first experiment we studied the effects of going without sleep from the ordinary rising time on one day until the ordinary retiring time the second night following, a period of from 60 to 65 hours.

The following tests were employed:

I. *Hand Dynamometer* (*Smedley*).—Three readings were taken for each hand at every test period. The subject chose an optimal setting for the instrument at the beginning of the experiment and this was kept constant throughout the work.

II. *Tapping.*—Up to the seventh or eighth day of the experiment, a lateral tapping instrument devised by one of the writers was employed. Certain unforeseen difficulties arose at that time, and, since there was not time in which to correct them, a shift was made to the ordinary tapping board and stylus. An electric clock counter was used as a recorder. The subject tapped for a period of three minutes with each hand, a two- or three-minute rest being given between the tapping with the right hand and that with the left. The usual precautions were taken as to the position of elbow, forearm, and wrist, and grip on the stylus.

III. *Aiming.*—The materials and apparatus recommended by Whipple[1] were employed. The metronome was set at 69 beats per minute. The subject went through the series of targets forward and then backward and forward and backward with each hand. That is, there were 40 thrusts for each hand.

IV. *Reading Letters.*—This test was adapted from one of Patrick and Gilbert's ('naming of letters'). A single prose passage of 1,000 letters (12-point type) was employed. The material was arranged in a column approximately 3.5 inches in width. The subject began reading the letters aloud from right to left in the lowest line and continued up the column—reading through each line from right to left. The experimenter kept track of the accuracy of the performance by means of a copy of the text, the letters of which were enlarged.

[1] Whipple, G. M., 'Manual of Mental and Physical Tests, Simpler Processes,' pp. 147–151. See the same work for a detailed description of the dynamometer and tapping tests.

V. *Mental Multiplication.*—Twelve two-place by two-place multiplications were performed at each sitting. The problems were chance arrangements of the digits from 3 to 9 inclusive. The problems were read to the subject by the experimenter. No graphic movements were permitted during the performance. Each incorrect digit in the results was counted an error.

The subjects were put through the tests once each day beginning from 11 to 26 days before the period of insomnia and continuing until the fourth or fifth day following the insomnia. The records for each day are given in Figs. I. to X., and the exact number of repetitions for each test can readily be observed in these figures. The testing was done at the same time of day for each subject. W. H. was tested between 6 and 7 P.M., S. H. between 7 and 8 P.M., and K. R. J. between 8 and 9 P.M. Because of increasing speed of performance in certain of the tests, the test periods dwindled from one hour or longer in the early stages of the experiment to from 30 to 45 minutes at the end. The testing during each period was practically continuous, and the tests were always given in the same order.[1]

Where both speed and accuracy were involved (reading letters and mental multiplication), the subjects were instructed to work as rapidly as possible without sacrificing accuracy. During the period of fore-exercise, the subjects had access to their previous records and strove constantly to better them.

So far as we can ascertain, the subjects were free from prejudice in regard to the outcome of the experiment.

The period of insomnia, as we have said, lasted from the usual rising time on one day until the usual retiring time on the second day following. That is, each subject went without sleep on two consecutive nights. During the daytime portion of the period, the subjects engaged in their ordinary activities. At night they did whatever would make their sleeplessness

[1] This order was dynamometer, reading letters, aiming, tapping, mental multiplication.

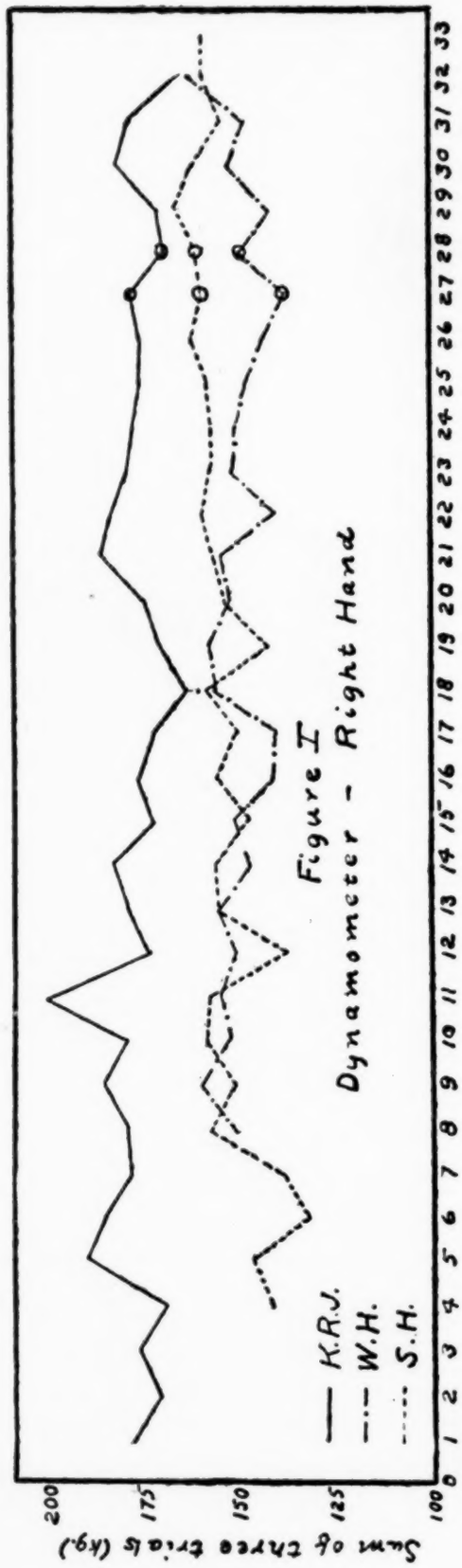
Figure I
Dynamometer - Right Hand
K.R.J.
W.H.
S.H.
Sum of three trials (kg.)
Days
100
125
150
175
200
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

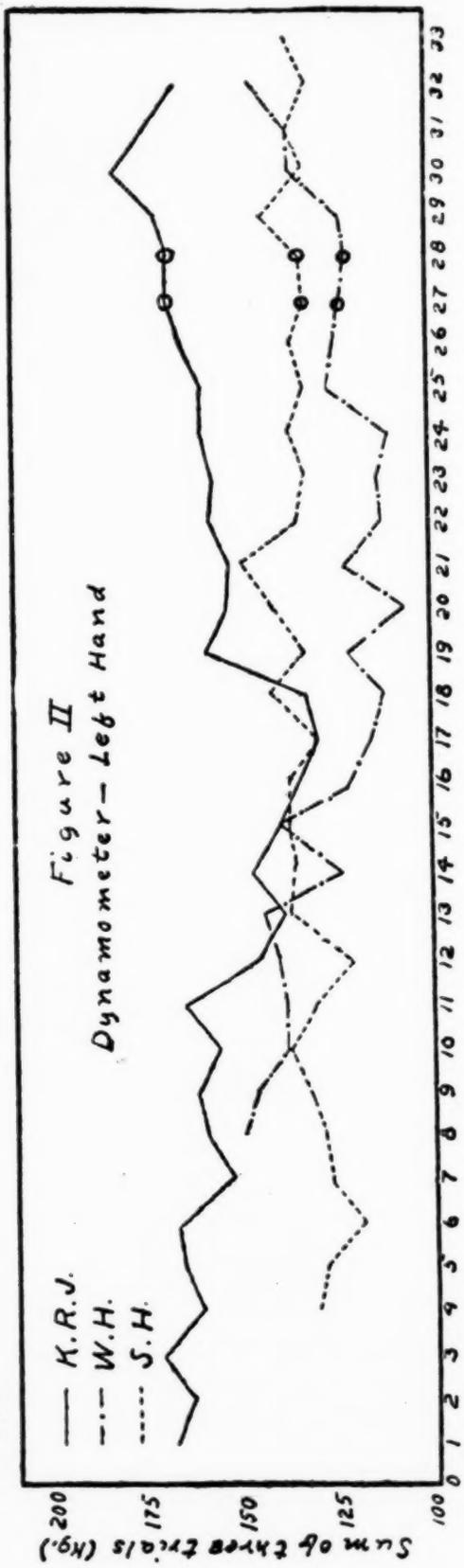
Figure II
Dynamometer - Left Hand
K.R.J.
W.H.
S.H.
Sum of three trials (kg.)
Days
100
125
150
175
200
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

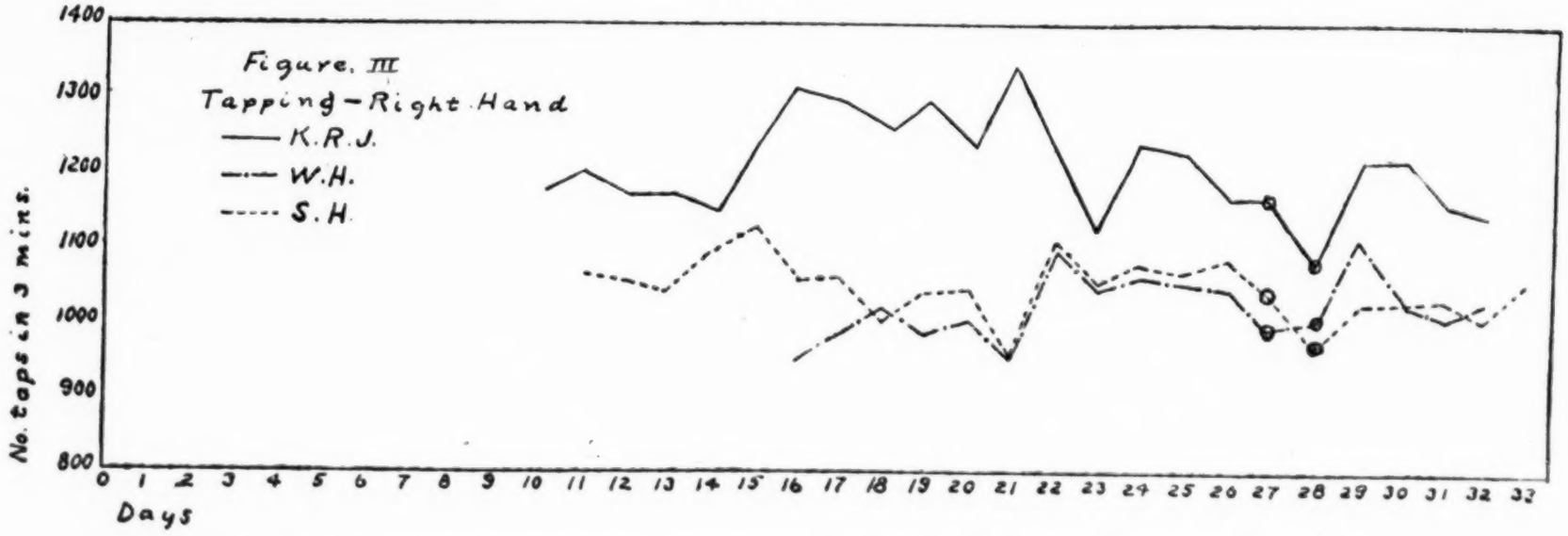
Figure. III
Tapping - Right Hand
K.R.J.
W.H.
S.H.
No. taps in 3 mins.
1400
1300
1200
1100
1000
900
800
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33
Days

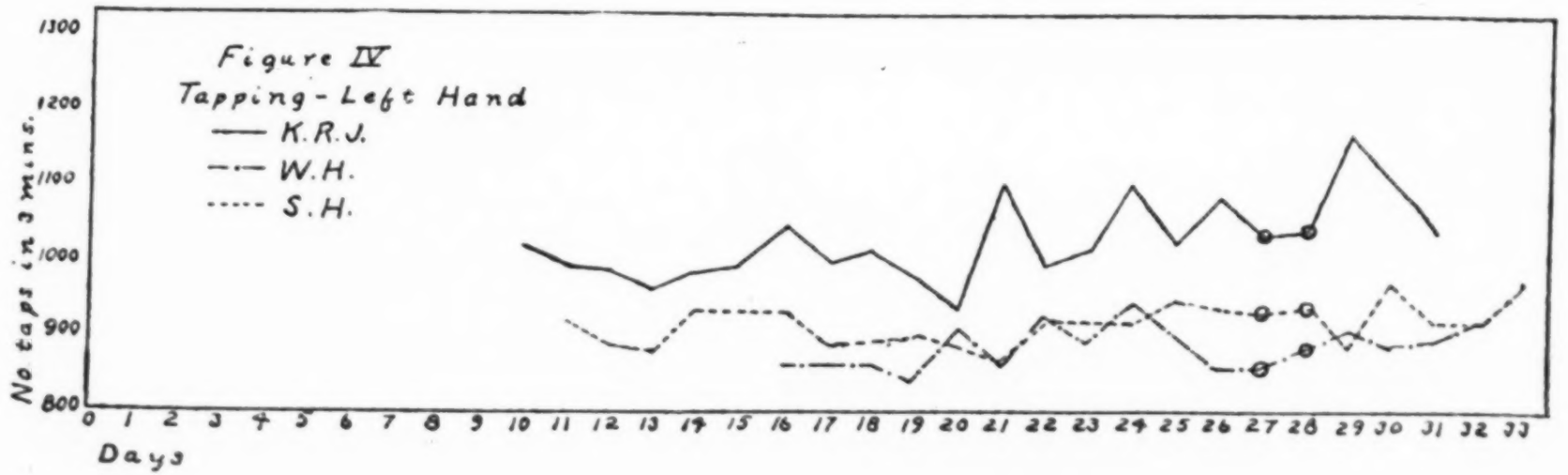
Figure IV
Tapping - Left Hand
K.R.J.
W.H.
S.H.
No taps in 3 mins.
1300
1200
1100
1000
900
800
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33
Days

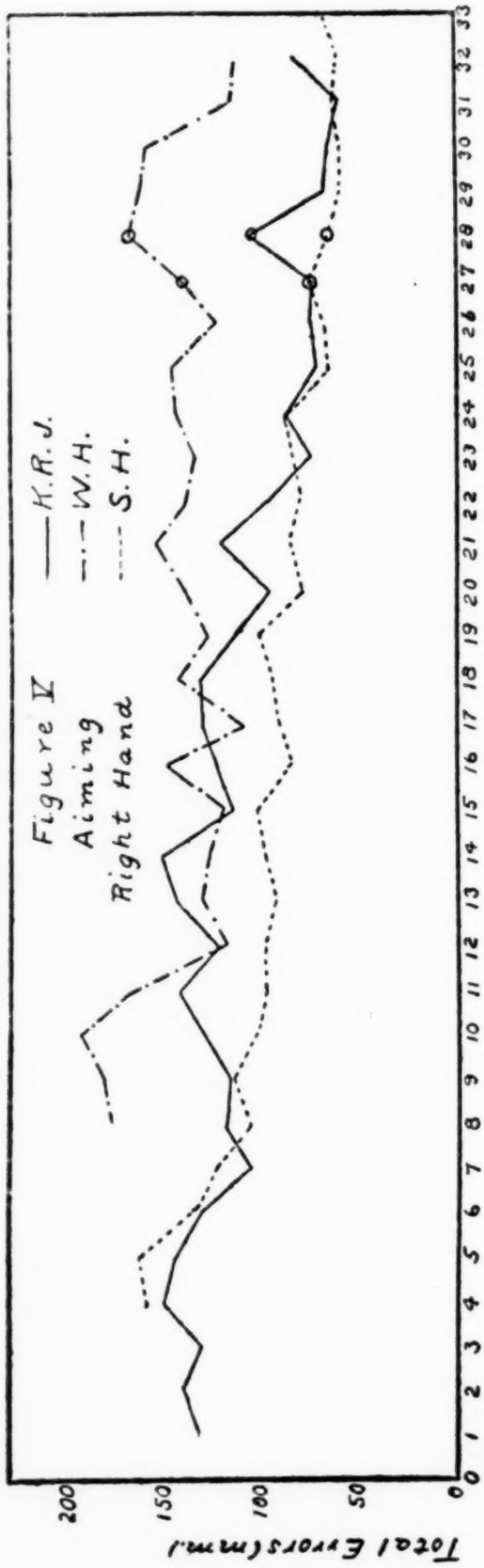
Figure V
Aiming
Right Hand
K.R.J.
W.H.
S.H.
Total Errors (mms.)
Days

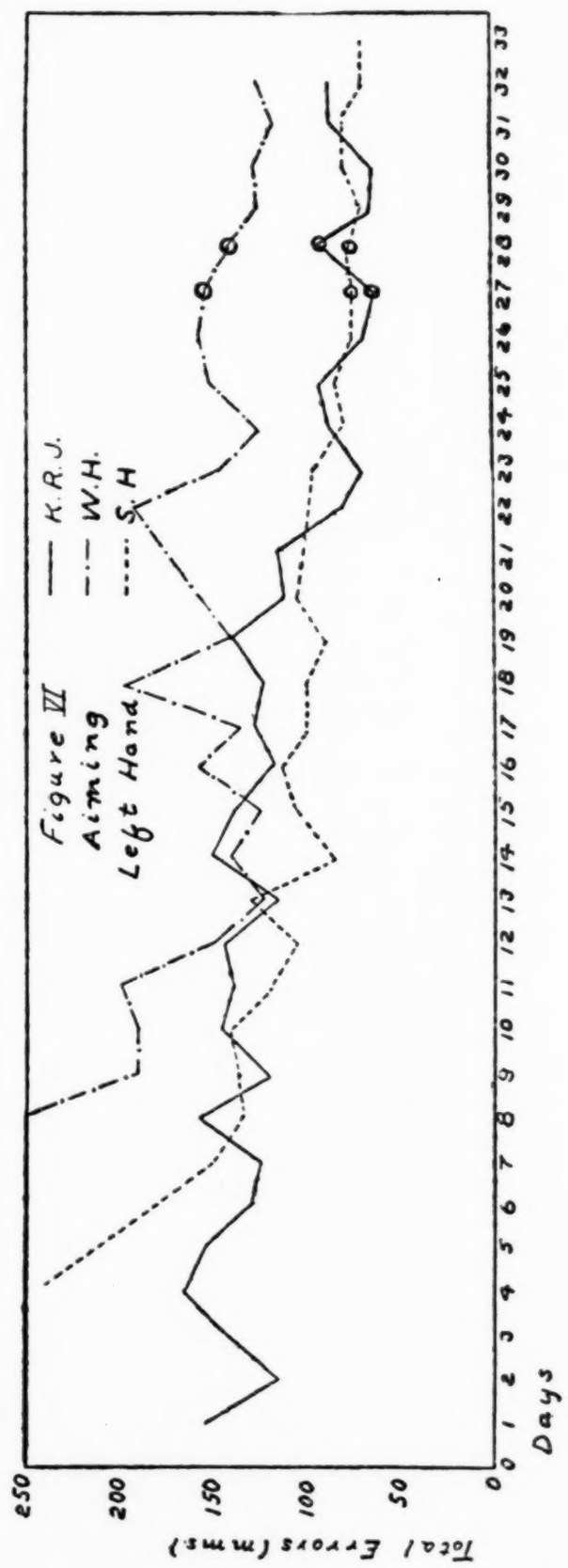
Figure VI
Aiming
Left Hand
K.R.J.
W.H.
S.H
Total Errors (mms.)
Days

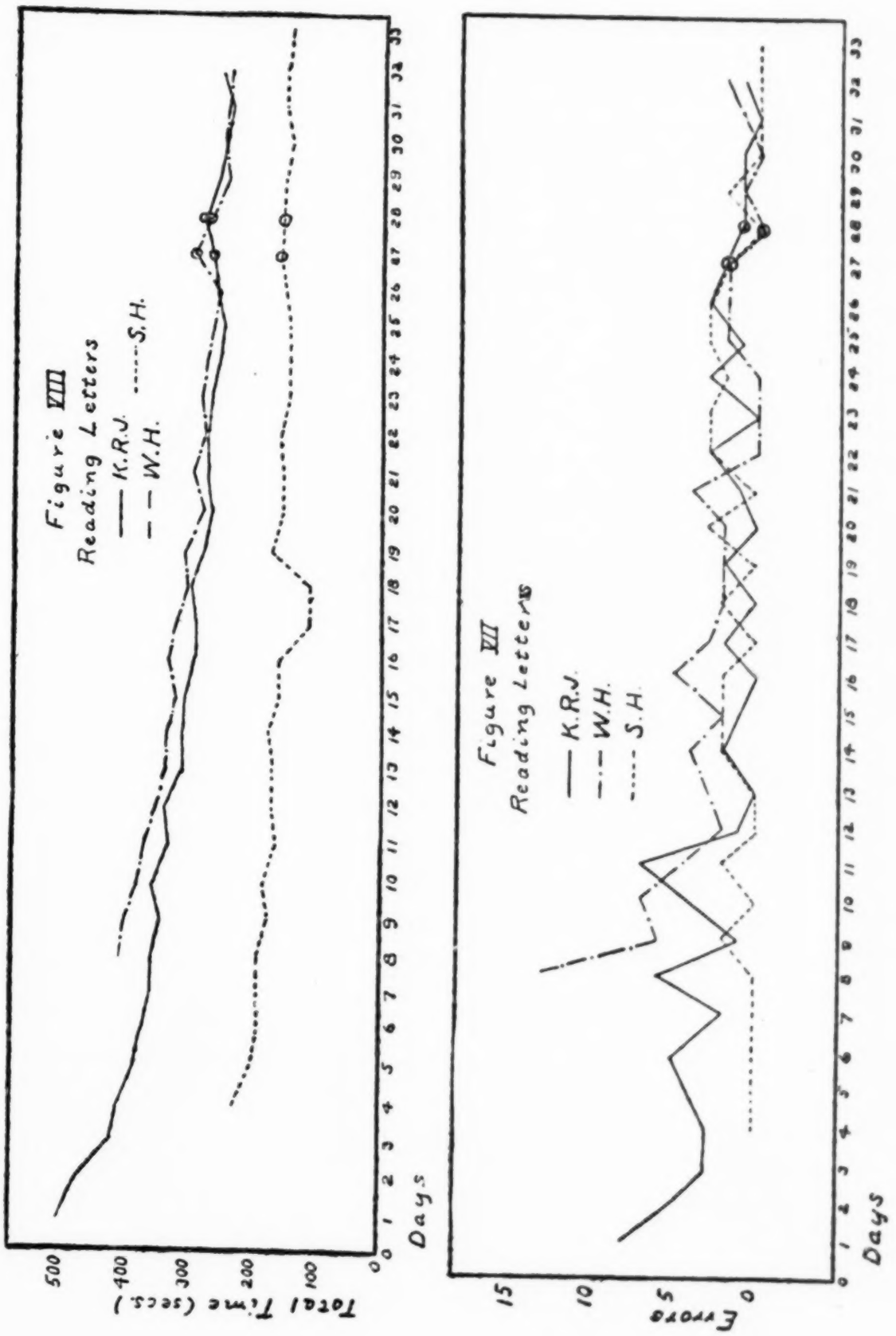
Figure XIII
Reading Letters
K.R.J.
W.H.
S.H.
Total Time (secs.)
Days
Figure XII
Reading Letters
K.R.J.
W.H.
S.H.
Errors
Days

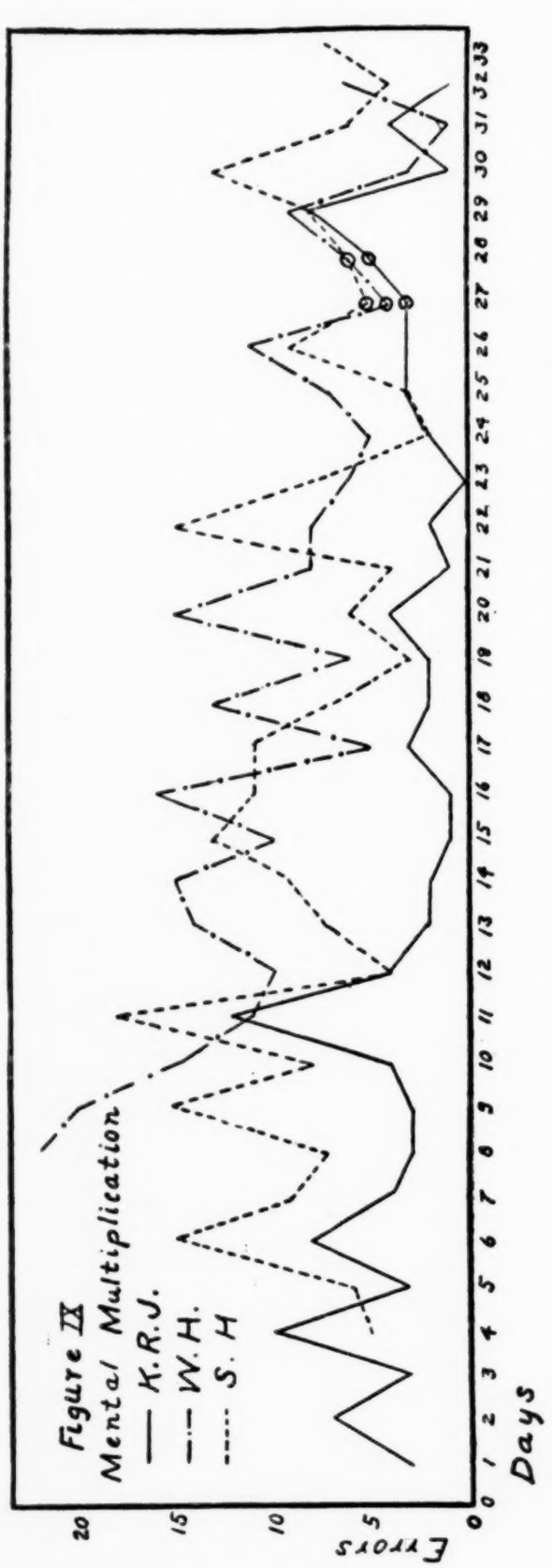
Figure IX
Mental Multiplication
— K.R.J.
-·- W.H.
---- S.H
Errors
Days

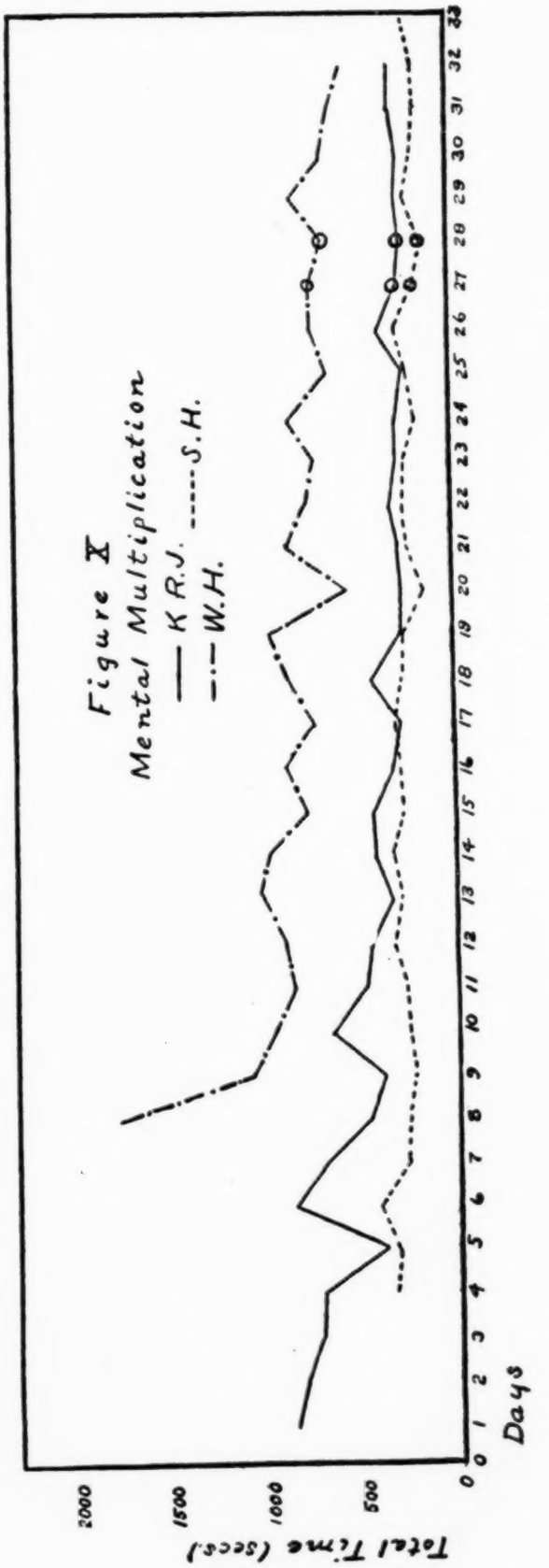
Figure X
Mental Multiplication
— K.R.J. ----- S.H.
-·- W.H.
Total Time (secs)
Days

most assured. This varied from reading and studying to walking about, attending the theater, and in one case, taking a short railroad journey.

Results

Figures I. to X. give the plotted records of the three subjects throughout the experiment. The points in the curves marked by circles represent performances during the insomnia. That is, records for day 27 are records obtained on the day following one sleepless night, and records for day 28 are those obtained on the day following the two consecutive sleepless nights. In the interests of more ready comparisons, the curves were placed on the graphs in such a way that the insomnia records of the different subjects coincide chronologically, although as a matter of fact they did not quite do so in the actual experiment.

The effects of the period of insomnia upon performance in these tests can be judged by inspecting the irregularities apparent in the curves of performance before, during, and after the loss of sleep. The irregularities in these curves before day 27, we may be sure, are no more due to unusual amounts of sleep than to undetermined variations in a normally uniform mode of life. Variations as great or consistent as these, then, may be caused by any one of the minor fluctuations of everyday life, and, unless a newly introduced condition, such as a period of insomnia, be accompanied by greater or more consistent variations, there are no grounds for estimating its effects.

The dynamometer curves show no effects from the insomnia. The right-hand curves of K. R. J. and S. H. in the tapping test show a possible loss due to the insomnia, but the evidence furnished by the right-hand curve of W. H. and by all three left-hand curves is negative. Perhaps the accuracy of K. R. J. and W. H. in the aiming test was affected deleteriously so far as their right hands were concerned, but the other curves for this test show no such effect. The results for reading letters are not very decisive. The fact that all three subjects improved slightly in accuracy during the

insomnia is not emphasized enough by our data to be taken seriously. All three of the error curves for mental multiplication rise during the insomnia. The time curves for the same test are apparently unaffected.

The main conclusion to be drawn from these curves is that the results of the tests were not affected by the insomnia in any marked or consistent manner. In so far as the curves show any general effect, they show a deleterious one. In only one (Fig. VII.) of ten sets of curves is there any evidence for increased efficiency during the insomnia. Our test results, then, would support the conclusions of Aschaffenburg, Roemer, and Patrick and Gilbert rather than those of Smith.

The fact remains, however, that most of our results are practically negative. This might be explained as due to the actual absence of any marked effects from the 60- to 65-hour insomnia, or to the fact that, feeling certain symptoms of lowered capacity, the subjects expended more effort on the tests during the period of sleeplessness. If this latter explanation is applicable to the almost negative character of our results, it might also be applied to the peculiar results of Smith. And it does not seem so highly incredible that her increased efficiency immediately following a loss of sleep may have been due to over-compensating through added effort for her lowered capacity. She claims that in at least one of her tests (the McDougall dotting machine) the possibility of variations due to changing amounts of effort was taken care of by the nature of the test. But the mere fact that the speed of performance is kept constant or that the test is exciting is, in our estimation, no guarantee that the subject will always exert a constant, in this case a maximum, amount of effort. Another factor which may have masked an actual loss of capacity in Smith and in our subjects is the increased interest in the general experimental situation which one might expect just at the time of actually going without sleep.

The qualitative effects of the insomnia upon our subjects were well marked. During the day following the first sleepless night, K. R. J. felt slightly nervous but otherwise

normal. Following the second sleepless night he reported a 'buzzing in the head,' he felt sleepy, and found it almost impossible during the morning to concentrate on a book that he tried to read. W. H. reported headache and burning eyes. On the evening following the first sleepless night, he said that he felt 'almost dead.' On the day following the second sleepless night, he was very nervous and jumpy. When spoken to, he answered rapidly and with absurd emphasis. S. H., from his reports, seems to have felt the insomnia least, but even with him there was a dazed feeling, very like mild intoxication, present after the first night without sleep and growing more pronounced throughout the sleepless period.

From these observations, it is evident that the loss of sleep did affect all of these subjects, and that at least one of them, W. H., was rather severely affected. It being admitted, then, that the loss of sleep had an effect upon our three subjects, the lack of more positive evidence in the test scores may mean that the tests happened to tap capacities comparatively uninfluenced by the insomnia, or that the changes in these capacities were compensated for by some such factor as extra effort. Because of the general nature of the qualitative effects noted by us, and also by other investigators, the second hypothesis seems the more reasonable. We know that many acts of muscular strength can be performed with what is apparently the same efficiency when we are tired as when we are fresh. But acts performed when we are tired usually involve new or additional muscles. This change in the muscular make-up of an act has as antecedents those 'sensations' which we think of as our tiredness. This is probably closely analogous to what happened to our subjects. As they felt the effects of insomnia increase, they may have exerted more effort on the different tests, which really amounts to saying that they began to alter their muscular or ideational means of producing constant or nearly constant objective results.

In some form or other the above theory has been offered in explanation of other failures to obtain markedly positive results with mental tests under conditions where important

changes in capacity must surely have taken place.[1] In the case of our experiment, the nervousness, the headache, the dazed condition, and the very noticeable disturbance of speech establish beyond a doubt the deleterious effects of a prolonged insomnia of which the test scores tell nothing. It is significant too, that such a function as mental multiplication is apparently as unmodified as the considerably simpler ones. If our explanation is the correct one, it points out an important limitation in the use of the usual tests for the detection of even relatively great disturbances. A desire to state the amount of such disturbances in purely objective language can be met only by test procedures of some refinement. Yet, even were such procedures readily available for application to a wide variety of functions, it is questionable whether the effects of loss of sleep, for instance, could be more vividly described than in terms of the visual hallucinations, the increased irritability, the emotional instability, the headaches, the nervousness, the dazed condition, and the disturbance of speech which this and other investigations have revealed. Suppose that we had been able to measure the degree of effort expended on mental multiplication on different days of the experiment, and suppose that we had found that efficiency during and after the insomnia was maintained only with x amount of additional effort. Those other results, obtained from what would generally be termed introspective and incidental observation, would still in our judgment be as important as any of our findings.

Our subjects reported that after a single night of sleep (average length, 8.5 hours) all detectable subjective consequences of the insomnia had disappeared. This agrees with Patrick and Gilbert, and with Smith. Our own objective results, in so far as they show any effects of loss of sleep, indicate that those effects did not persist for long after the insomnia. Here we are more in agreement with Patrick and Gilbert than with Smith.

[1] See Morgan's discussion and demonstration of the need for something other than the conventional measures of efficiency in terms of production. *Arch. of Psychol.*, 1916, No. 35.

THE EFFECT OF CHANGED DATA UPON REASONING[1]

BY EDWARD L. THORNDIKE

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The older psychology, perpetuated in current educational doctrines and practices, regarded reasoning as a force largely independent of associative habits, which worked back to correct or oppose them. Our present psychology finds that the mind is ruled by habit throughout, the correction or opposition being of certain more simple, thoughtless and coarse habits, by others which are more elaborate, selective, and abstract. It defines reasoning as the organization and coöperation of habits rather than as a special activity above their level; and expects to find 'reasoning' and habit or association working together in almost every act of thought.

One interesting and rather important consequence of this view is the theorem that "Any disturbance whatsoever in the concrete particulars reasoned about will interfere somewhat with the reasoning, making it less correct or slower or both." It is the purpose of this article to give illustrations of and evidence for this theorem drawn from a simple experiment which can be easily given to any class, and is perhaps deserving of inclusion in a list of group experiments in psychology.

Consider the two sets of tasks in algebra, printed below. Each pair of tasks demands the application of the same principle, but the concrete situation in the one case is that with which our ordinary associative habits have been made, whereas in the other case the concrete particulars are somewhat altered. The alterations vary from slight ones, such as using p instead of x, or $b_1 + b_2$ instead of $x + y$, to a

[1] The experiments reported here were part of an investigation made possible by a grant from the Commonwealth Fund.

change from a very customary statement of a set of relations to a very rare statement, as in No. 9.

Ha.	Ch.
Habitual	*First Changed Form*
1. What is the square of $x + y$?	1. What is the square of $b_1 + b_2$?
2. What is the square of a^2x^3?	2. What is the square of $r_1^3 r_{11}^2$?
3. Simplify $4ac + \left(\frac{b^2}{c^2} \times \frac{c}{d^2} \times \frac{d^3}{b}\right)$.	3. Simplify $5a^2b + (b^2 \div c^2)\,(c^2 \div d^2)\,(d^3 \div b)$.
4. What are the factors of $x^2 - y^2$?	4. What are the factors of $1/x^2 - 1/y^2$?
5. Multiply x^a by x^b.	5. Multiply 4^a by 4^b.
6. Simplify $ac - [a(b + c)]$.	6. Simplify $p_1p_3 - [p_1(p_2 - p_3)]$.
7. Solve $x + y + z = 15$, $2x + y + 3z = 22$, $x + 2y + z = 25$.	7. Solve $c_1 + c_2 + c_3 = 15$, $2c_1 + c_2 - 3c_3 = 22$, $c_1 + 2c_2 + c_3 = 25$.
8. $e^2 + ef = \frac{g}{x}$. What does x equal?	8. $e^2 + ef = \frac{g}{p}$. What does p equal?
9. There are two numbers. The first number plus 3 times the second number equals 7. The first number plus 5 times the second number equals 11. What are the numbers?	9. $y = ax + b$. When $x = 3$, $y = 7$. When $x = 5$, $y = 11$. What does a equal? What does b equal?

The probability of error and delay in the necessary reasoning is clearly increased when the task is of the habitual sort (Ha.) rather than the changed sort (Ch.).

The subjects which I used were ninety-seven graduate students, divided at random into three groups of 34, 32 and

31. Group *A* did tasks 1 and 7 of Ha., tasks 4, 5, 6, 8 and 9 of Ch. and two other tasks as follows: "What is the square of $k^2p^3?$" and "Simplify $3ab + [b^2cd^3/bc^2d^2]$." Group *B* did tasks 3, 6 and 9 of Ha., tasks 1 and 2 of Ch. and four other tasks as follows: "What are the factors of $x - y?$" "Multiply x^4 by x^3." "Solve $x + y = 13$, $y + z = 12$, $x + z = 5$." "$e^2 + ex = g/a$. What does a equal?" Group *C* did tasks 2, 4, 5 and 8 of Ha., tasks 3 and 7 of Ch. and three other tasks as follows: "Multiply $(b_1 + b_2)$ by $(b_2 - b_1)$." "Simplify $mn - [2n(m - p)]$." "The cost of a season ticket is b dollars plus k times the cost of a month ticket. When the cost of a month ticket is $6, the cost of a season ticket is $15. When the cost of a month ticket is $15, the cost of a season ticket is $33. What is the value of b? What is the value of $k?$" [1]

The relative abilities of the groups may be estimated by comparing the scores in tasks estimated to be of equal difficulty, such as:

$(x + y)^2 = ?$ and Solve $e^2 + ef = g/p$ for p, for *A*,

$x^4 \times x^3 = ?$ and Solve $e^2 + ex = g/a$ for a, for *B*,

and Factor $(x^2 - y^2)$ and Solve $e^2 + ef = g/x$ for x, for *C*. Using these combinations, the percent of right answers was 71 for Group *A*, 84 for Group *B*, and 63 for Group *C*. There is thus an approximate equalization of ability of subjects between the *Habitual* and the *Changed* series.[2]

All three groups worked by the same time-schedule, which was: "0, Begin; 1 min. 30 sec., even if you have not finished 1, begin on 2; 2.30, even if you have not finished 2, begin on 3; 4.30, even if you have not finished 3, begin on 4;" and so on with 6.30 for proceeding to 5; 8.30 for 6; 9.30 for 7; 12.30 for 8; 13.30 for 9; 16.30 to stop. The subject could thus use time saved on one task for another, absolute uni-

[1] The fractions in the above and throughout were all presented in the usual 'horizontal' form, e.g. $\frac{g}{a}$.

[2] The *B* group is probably really somewhat more superior than these figures show, and the differences shown in Table I. below are somewhat greater than would be the case with perfect equalization. However, there can be no doubt that a substantial difference would remain.

formity being had only in respect to the 16 min. 30 sec. spent on the entire set of nine tasks.

The percents wrong or incomplete for the nine tasks where customary associations were favored and for the nine where some change was made were, in order:

TABLE I

	Customary	Changed	Nature of Change
1.....	6	28	$(x + y)$ to $(b_1 + b_2)$
2.....	34	47	a^2x^3 to $r_1^3r_{11}^2$
3.....	37½	64½	$-$ to $\div$ form
4.....	22	41	$x^2 - y^2$ to $\frac{1}{x^2} - \frac{1}{y^2}$
5.....	55	70½	x^2 to 4^2
6.....	25	53	ac, etc., to p_1p_3, etc.
7.....	62	61	xyz to $c_1c_2c_3$
8.....	52	53	x to p
9.....	16	70	form of problem

In all but two cases (7 and 8) there is a substantial interference with thought by the change. In No. 9 the amount of novelty introduced is much greater than in the other cases, and is perhaps more truly called a change in the principles and operations used than a change in the concrete particulars reasoned about. In No. 6 the use of a minus sign in place of the plus may have added a little difficulty over and above the change from literal to subscript distinctions.

We may now examine certain facts from the accessory tasks which were used partly to conceal the special point of the experiment from the participants and partly to secure additional data.

"Multiply $(b_1 + b_2)$ by $(b_2 - b_1)$" is not strictly comparable with "What is the square of $x + y$?" but the rise in errors from 6 percent for the latter (Group *A*) to 61 percent for the former (Group *C*) would probably have been much less if "Multiply $(x + y)$ by $(y - x)$" had been used.

The change from "What is the square of a^2x^3?" (Group *C*) to "What is the square of k^2p^3?" (Group *A*) raises the errors and omissions from 34 percent to 47 percent.

Simplify $3ab + [b^2cd^3/bc^2d^2]$ produces more errors than the same task when arranged in $+ (\times \times)$ or in $+ (\div \div)$ form (71 percent in Group *A*). This may seem to be in contradiction to our general theorem, but an inspection of the errors makes it probable that the parentheses were a protection against two habits, one of canceling upper and lower numbers seen in proximity, the other of clearing of fractions any expression that contains one. The former leads to canceling the b of $3ab$ with the b of bc^2d^2, which is done by one of the thirty-four subjects. The latter adds an operation in which errors are made by four of the subjects, and makes the needed cancellations more difficult. It also encourages the error of dropping out the common denominator, which is done by four more, and permits the worker to feel that he has done his duty by the task, though he has not cancelled. Finally, there are further erroneous manipulations of the denominator (as in $(3ab + b^2cd^3)/bc^2d^2$) from which the parentheses and signs of multiplication and division were perhaps a protection in the other forms of the task.

The question "What are the factors of $x - y$?" evokes responses which show that when a novel situation is met, one's associative habits do not retire while reason attacks. On the contrary, the forces of habit are nowhere more evident than in the treatment of novel situations. Which habits will act depends on which elements or features of the situation are given weight and upon the amount of weight given to each. With our subjects, the features that this is the difference not of two particular numbers but of any number from any number and that 'any number' is as truly the square of '$\sqrt{\text{any number}}$' as '(any number)^2' is of 'any number,' are usually neglected or underweighted, there being only four answers of

$$(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) \text{ or } (x^{1/2} + y^{1/2})(x^{1/2} - y^{1/2})$$

The underweighting of these features permits the habits of not factoring prime numbers in arithmetic, and of not factoring expressions like $x - y$ in algebra, to act acceptably, answers of $1(x - y)$ being the commonest given (11 out of 32).

"Multiply x^4 by x^3" is, consistently with our general theorem, easier than x^a by x^b.

On the whole, it seems certain that even such slight changes as from the customary a, b, x and y, to k and p or to p_1, p_2 and p_3 or to p_1, p_{11} and p_{111}, impede thought, and that the general theorem does hold that "Any disturbance whatsoever in the concrete particulars reasoned about will interfere with reasoning."

THE EFFECT ON FOVEAL VISION OF BRIGHT (AND DARK) SURROUNDINGS. V

BY ELLIOT Q. ADAMS AND PERCY W. COBB,

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A study of the variation of the difference-threshold with independent changes in the brightness of the test field and of the surroundings has been reported in an earlier paper.[1] In that paper there were deduced, on the basis of a physico-chemical explanation of contrast, certain theoretical considerations which proved to be in only limited agreement with the results of observation. It has since been found possible with a few simple, and qualitatively[2] plausible, assumptions as to the nature of the nerve processes of the retina, to derive a theoretical expression for the variation of the difference-threshold which is in good agreement with the experimental results.

Assumptions

The assumptions made are:

1. Over the range of ordinary photometric work, with complete adaptation to a field of constant and uniform brightness, the fractional difference-threshold reaches a value which is independent of the brightness, and not greater than that for any non-uniform field.

2. With a non-uniform field, the state of adaptation at any point is that which would have resulted from a uniform field of brightness equal to a weighted[3] quadratic mean of the brightness of the different parts of the field.

[1] P. W. Cobb, J. Exp. Psych., *1* (6), 540–566 (Dec., 1916).

[2] Exact accuracy of the postulates is not claimed. The accuracy possible in determinations of threshold does not appear to warrant refinement of the assumptions made.

[3] 'Weighted' to take account of the relative areas of the different parts of the field, and their visual importance with reference to the test field. The *arithmetic* mean was tried but the formula so derived did not agree so well with the experimental results.

3. The response of a fiber of the optic nerve consists of a series of impulses[1, 2] whose frequency increases, at constant adaptation, with the brightness at the corresponding point of the field.

4. The interval between impulses consists of two parts, a constant 'absolute refractory period,' and a 'relative refractory period' which varies inversely with the brightness at the corresponding point of the field, *i.e.*, after the end of the absolute refractory period following one impulse the sensitiveness (the reciprocal of the least effective brightness) increases linearly with time until its product with brightness reaches a certain critical value, whereupon another impulse follows.

5. Equal differences in frequency of response are equally perceptible.

Constant Adaptation

For simplicity consider first the variation of the fractional difference-threshold for a 'test field' as a function of the variable brightness of the test field. This condition will obtain *approximately* when a test field of the smallest practicable size is viewed against constant surroundings.

The interval between impulses will be

$$t = t_\infty + \frac{kt_\infty}{B}, \qquad (1)$$

where t is the interval; t_∞ the absolute refractory period, assumed constant; B the test-field brightness and k a constant such that kt_∞/B is the relative refractory period. It can be deduced from postulates 1 and 5 that k will vary proportionally with the adaptation brightness, B_1, over the range within which complete adaptation is possible; and it will be shown below that $k = B_1$.

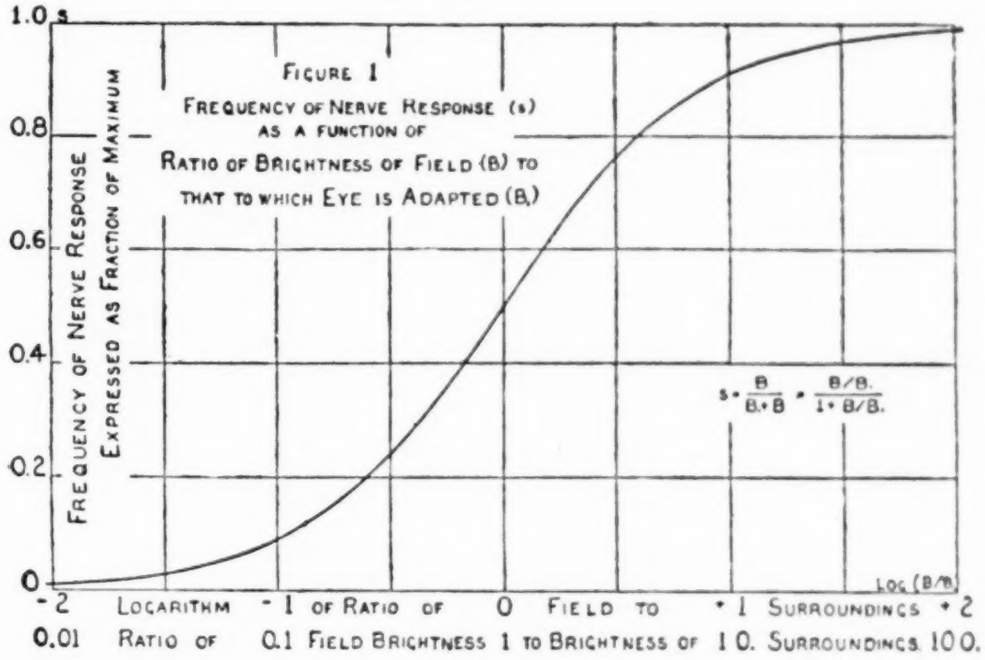
[1] Keith Lucas, 'The Conduction of the Nervous Impulse,' London (1917), p. 9.

[2] L. T. Troland, *J. Opt. Soc. Am.*, *4*, 160 (1920).

The *frequency* of the impulses will be $1/t$, and the ratio of the frequency to the maximum possible, $1/t_\infty$, will be

$$s = \frac{t_\infty}{t} = \frac{1/t}{1/t_\infty} = \frac{1}{1 + (k/B)} = \frac{B}{B + k}. \qquad (2)^6$$

This ratio of frequencies is shown, in Fig. 1, plotted against log (B/k), the logarithm of the brightness of the test field (in terms of $k = B_1$).


1.0 s
0.8
0.6
0.4
0.2
0
FREQUENCY OF NERVE RESPONSE EXPRESSED AS FRACTION OF MAXIMUM
FIGURE 1
FREQUENCY OF NERVE RESPONSE (s) AS A FUNCTION OF
RATIO OF BRIGHTNESS OF FIELD (B) TO THAT TO WHICH EYE IS ADAPTED (B1)
s = B/(B+B1) = (B/B1)/(1+B/B1)
log (B/B1)
-2 LOGARITHM -1 OF RATIO OF 0 FIELD TO +1 SURROUNDINGS +2
0.01 RATIO OF 0.1 FIELD BRIGHTNESS 1 TO BRIGHTNESS OF 10. SURROUNDINGS 100.

By assumption (5), since t_∞ is constant, the least perceptible change in brightness, ΔB, will be that associated with a constant small change Δs in the ratio of frequencies.

Differentiating (2)

$$ds = \frac{(B + k)dB - B\,dB}{(B + k)^2} = \frac{k\,dB}{(B + k)^2} = \frac{kB}{(B + k)^2}\frac{dB}{B}. \qquad (3)$$

Regarding Δs and ΔB as infinitesimal changes, the least perceptible fractional change in B, *i.e.*, the fractional difference threshold, is related to Δs as follows:

$$\frac{dB}{B} = \frac{(B + k)^2}{kB}\,ds = \left(\frac{B}{k} + 2 + \frac{k}{B}\right)ds. \qquad (4)$$

Differentiating the coefficient of ds in (4) and equating to zero to determine the value of B corresponding to minimum threshold,

$$\frac{d}{dB}\frac{(dB)/B}{ds} = \frac{d}{dB}\left(\frac{B}{k} + 2 + \frac{k}{B}\right) = \frac{1}{k} - \frac{k}{B^2} = 0. \qquad (5)$$

Whence $k = B$, but since by assumption (1) the fractional difference threshold is a minimum when the field is uniform, that is when $B = B_1$,

$$k = B_1. \qquad (6)$$

Substituting in (4) and writing ΔB and Δs for dB and ds:

$$\frac{\Delta B}{B} = \left(\frac{B}{B_1} + 2 + \frac{B_1}{B}\right)\Delta s. \qquad (7)$$

The coefficient of Δs in (7) is shown in Fig. 2, plotted against log B/B_1, the logarithm of the ratio of the brightness of the (infinitesimal) test field to that of the surroundings.

Effect of Test Field on Adaptation.

If account be taken of the effect of the brightness of the test field on the state of adaptation, equation (7) will still hold, provided that by B_1 is meant not the brightness of the surroundings B', but the quadratic mean brightness,

$$B_1 = \sqrt{\alpha B^2 + (1 - \alpha)B'^2}, \qquad (8)$$

[1] It is of interest to point out that equation (9) of the earlier paper (1),

$$L = \frac{WB}{S - W}$$

becomes, on solving for W/S

$$\frac{W}{S} = \frac{L}{L + B}$$

or in the notation of this paper, if W/S be identified with s:

$$s = \frac{B}{B + k},$$

which is identical in form with equation (2) above.

It follows that the assumptions of the former paper would, with the addition of assumption (1) of the present article, give the same mathematical results as those derived above.

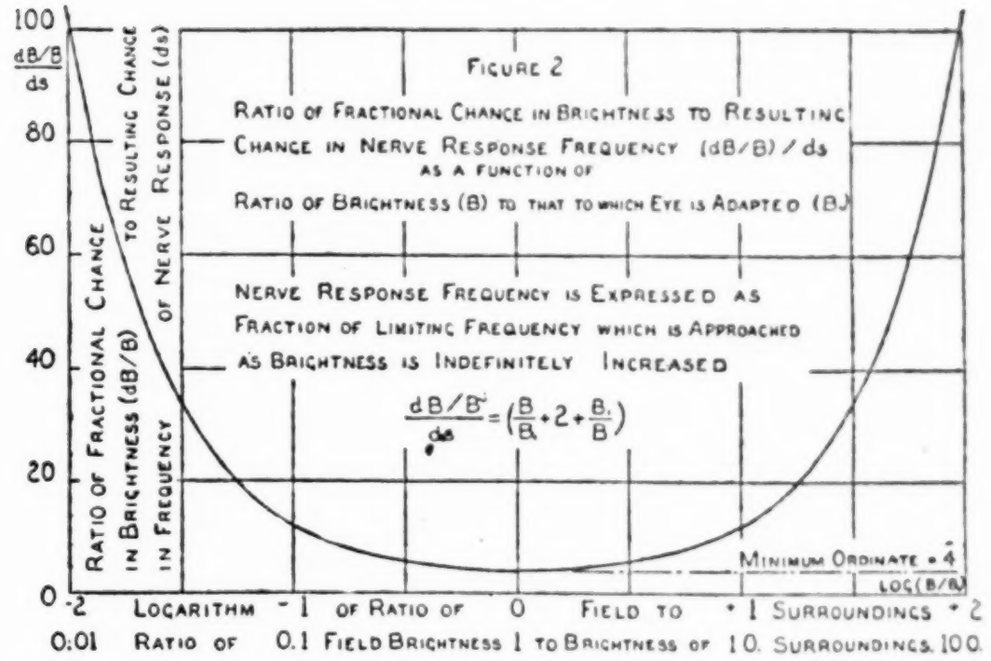


FIGURE 2
RATIO OF FRACTIONAL CHANGE IN BRIGHTNESS TO RESULTING CHANGE IN NERVE RESPONSE FREQUENCY (dB/B) / ds AS A FUNCTION OF RATIO OF BRIGHTNESS (B) TO THAT TO WHICH EYE IS ADAPTED (Bo)
NERVE RESPONSE FREQUENCY IS EXPRESSED AS FRACTION OF LIMITING FREQUENCY WHICH IS APPROACHED AS BRIGHTNESS IS INDEFINITELY INCREASED
$\frac{dB/B}{ds} = \left(\frac{B}{B_o} + 2 + \frac{B_o}{B}\right)$
RATIO OF FRACTIONAL CHANGE IN BRIGHTNESS (dB/B) TO RESULTING CHANGE IN FREQUENCY OF NERVE RESPONSE (ds)
100
80
60
40
20
0
MINIMUM ORDINATE = 4
LOG(B/Bo)
-2 LOGARITHM -1 OF RATIO OF 0 FIELD TO +1 SURROUNDINGS +2
0.01 RATIO OF 0.1 FIELD BRIGHTNESS 1 TO BRIGHTNESS OF 10. SURROUNDINGS 100.

where α is a coefficient, to be determined empirically, expressing the relative effectiveness of test field and surroundings on adaptation.

Comparison with Experimental Results.

For comparison with the equations (7) and (8), Table I. gives the observed fractional threshold,[1] as a function of the

Table I

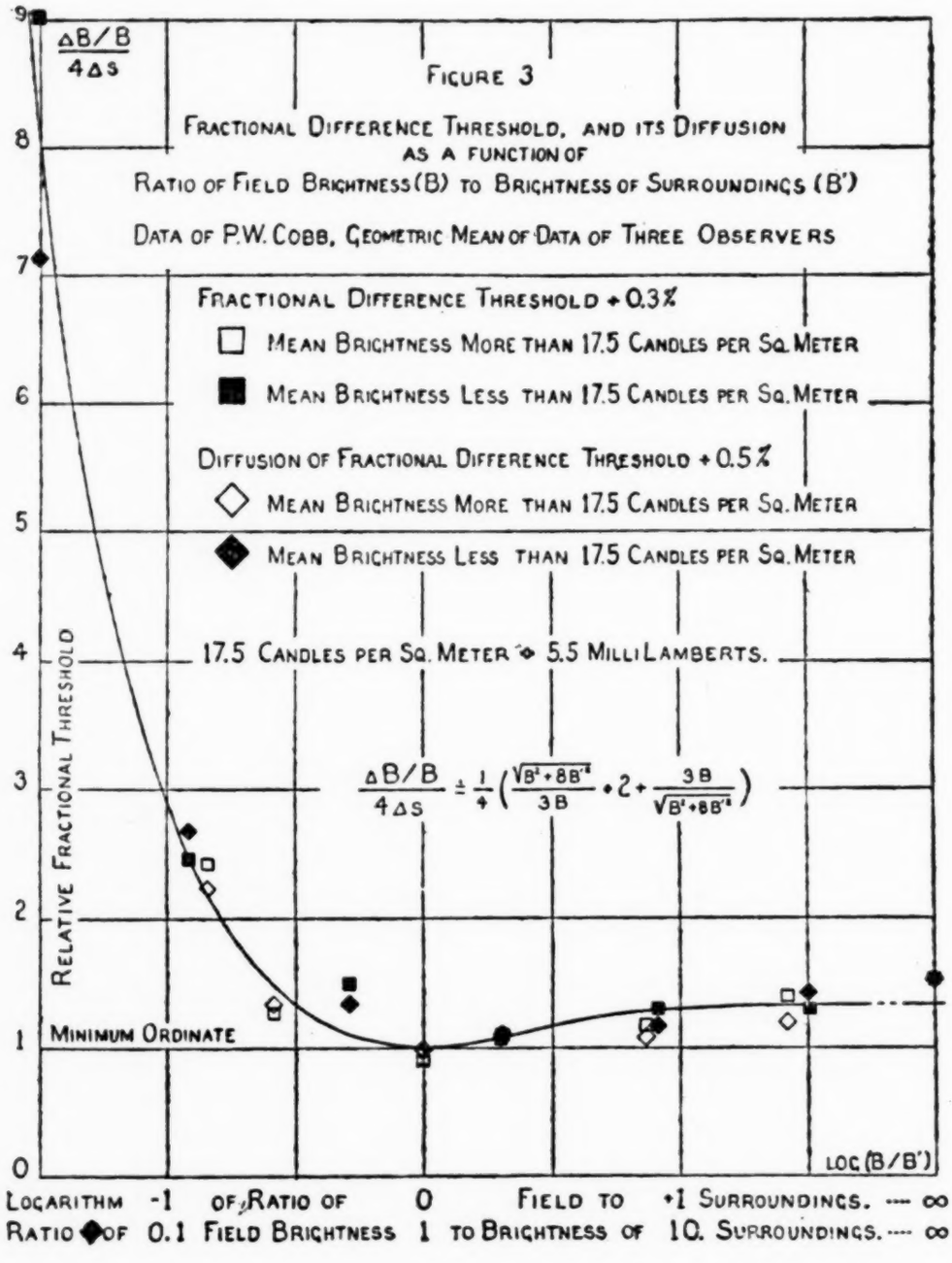
Difference Threshold and Diffusion, Measured as Fractional Changes in One Half of a Test Field of Angular Dimensions 2.0° by 2.6°

	Brightness		Fractional Difference			Brightness		Fractional Difference	
	Test Field	Surroundings	Threshold	Diffusion		Test Field	Surroundings	Threshold	Diffusion
a	436	17.3	0.42%	0.60%					
b	128	17.3	0.35	0.54	*u*	17.5	121	0.73%	1.12%
c	35.3	17.3	0.32	0.54	*v*	17.6	46.0	0.38	0.67
o	17.6	17.3	0.27	0.49	(*o*	17.6	17.3	0.27	0.49)
d	8.92	17.3	0.45	0.67	*w*	17.6	8.64	0.33	0.55
e	2.11	17.3	0.74	1.34	*x*	17.6	2.15	0.39	0.61
f	0.540	17.3	2.71	3.57	*y*	17.6	0.547	0.39	0.71
g	0.0	17.3	∞	∞	*z*	17.6	0.0	0.46	0.76

Brightness is expressed in candles per square meter.

[1] Geometric mean of values for three observers. Quoted from Table V., p. 547 article cited in Footnote 1.

brightness of the test field, and of that of the surroundings. These data are shown by points in Fig. 3, while the line represents the coefficient of Δs in equation (7), when the


ΔB/B / 4Δs
FIGURE 3
FRACTIONAL DIFFERENCE THRESHOLD, AND ITS DIFFUSION AS A FUNCTION OF
RATIO OF FIELD BRIGHTNESS (B) TO BRIGHTNESS OF SURROUNDINGS (B')
DATA OF P.W. COBB, GEOMETRIC MEAN OF DATA OF THREE OBSERVERS
FRACTIONAL DIFFERENCE THRESHOLD ± 0.3%
MEAN BRIGHTNESS MORE THAN 17.5 CANDLES PER SQ. METER
MEAN BRIGHTNESS LESS THAN 17.5 CANDLES PER SQ. METER
DIFFUSION OF FRACTIONAL DIFFERENCE THRESHOLD ± 0.5%
MEAN BRIGHTNESS MORE THAN 17.5 CANDLES PER SQ. METER
MEAN BRIGHTNESS LESS THAN 17.5 CANDLES PER SQ. METER
17.5 CANDLES PER SQ. METER = 5.5 MILLILAMBERTS.
ΔB/B / 4Δs ± 1/4 (√(B²+8B'²) / 3B + 2 + 3B / √(B²+8B'²))
RELATIVE FRACTIONAL THRESHOLD
MINIMUM ORDINATE
LOG (B/B')
LOGARITHM OF RATIO OF FIELD BRIGHTNESS TO BRIGHTNESS OF SURROUNDINGS.
RATIO 0.1 1 10. ∞
-1 0 +1 ∞
0 1 2 3 4 5 6 7 8 9

coefficient α, of equation (8) is given the value $\alpha = 1/9$. To reduce the three functions to a common basis, namely a minimum ordinate of unity, the values of the difference-

threshold have been divided by 0.3 per cent., those of the diffusion by 0.5 per cent., and the coefficient of Δs in (7), by 4. The thresholds are indicated by squares the diffusion values by 'diamonds', Black squares indicate results obtained when the mean brightness was less than 17.5 candles per square meter, white squares when the mean brightness was greater. It will be seen by comparing the number of light and dark squares above and below the curve that increase in the general level of brightness causes a slight but apparently definite reduction in the thresholds. This difference represents the degree in which the assumptions (in particular assumption (1) in the opinion of the writers) fail to take account of the facts.

FIG. 1

FIG. 2

With a larger test field the loss in accuracy with dark surroundings would be less pronounced.

CONCLUSION

Based on the assumption that visual impressions are transmitted along each fiber of the optic nerve by a series of impulses whose effect depends only on their frequency (the All-or-None hypothesis of Keith Lucas[4, 5]) quantitative expressions for the effect of light and dark surroundings on vision have been derived, which for foveal vision, with empirically selected values of the coefficient for the relative effect of different parts of the field, accord well with observations of difference threshold, and of its 'diffusion.'

Further experimental work will be needed to establish the way in which the coefficient before mentioned is related to the size and location of the test field, the extent to which the assumptions on which the formulas are based are in accord with fact, and the range of conditions over which they continue to hold.

NELA RESEARCH LABORATORIES,
NELA PARK, CLEVELAND, OHIO,
March, 1922

A NEW LABORATORY AND CLINIC PERIMETER

BY C. E. FERREE AND G. RAND,

Bryn Mawr College

This apparatus was devised in response to a request from a committee appointed by the American Ophthalmological Society to work out a better standardization of the illumination of perimeters and test charts. The request was for a feasible means of illuminating the perimeter arm with light of a good intensity and quality, so that every point on the arm in any meridian in which it may be placed shall receive equal intensities of light. Intensity and quality of illumination, however, are only two of the factors which influence the results of the perimetric determination. In devising the instrument described in this paper it has been our purpose to provide a control also of other factors which are of importance to the work of the office and clinic. The perimeter devised and to be described in this paper has been adopted by the Committee as the standard instrument for office and clinic work.

The variable factors which influence the apparent limits of color sensitivity are, so far as we have been able to discover, the wave-length and purity of the stimulus, the intensity of the stimulus and the visual angle, length of exposure of the eye, the method of exposure (moving or stationary stimulus), accuracy and steadiness of fixation, the intensity of the general illumination of the retina and its state of adaptation, breadth of pupil, and the brightness of the preëxposure and of the background or surrounding field. The most important of these from the standpoint of the office or clinic are perhaps the intensity of the stimulus, the brightness of the preëxposure and the surrounding field, the intensity of the general illumination, and the accuracy and steadiness of fixation.

Perhaps errors in refraction should be included in the above list of factors. They differ from those factors, however,

in that they are a source only, or mainly at least, of differences in breadth of field between observers, not for the same observer at different times. They are not therefore a serious source of trouble in the use of perimetry to check up the advance or recession of a given pathological condition, but they are confusing in diagnosis. We have no certain means of telling, for example, how much of the results in any given case of high myopia is due to the refractive condition and how much if any is pathological. To add to our knowledge on this point we are now conducting an investigation to determine the effect of various amounts and kinds of refractive errors on the results of field taking. We can not help but feel, as is stated and discussed in the paper, that a provision should be made in our practice of field taking to include one set of results under the correcting glasses, even though it means either working under conditions which give a narrow field or being content with results which do not include the wider portions of the temporal field.

1. *Intensity of Stimulus.*—By a sufficiently wide variation of this factor alone, the zones of color sensitivity may be made to have almost any breadth within the limits of the field of vision, to differ radically in shape, and even to change or reverse their order of ranking as to breadth. When pigment surfaces are used as stimuli the illumination of the perimeter arm determines the intensity of the stimulus light. Two methods are proposed for securing an even illumination of the stimulus at every point on the perimeter arm and of reproducing this illumination from time to time.

Method 1.—When the source of light is inlaid in the surface of the arm or its continuation, the illumination on this surface will be equal for approximately 180° on either side of the source. The value of this illumination at every point will be equal to the normal flux of light from the luminous surface divided by four times the square of the radius of curvature of the perimeter arm, or four times the square of the distance of the eye from the perimeter arm. A perimeter embodying

this principle of illumination is being constructed in the following way. A lamp house is fastened on the continuation of a 90° arm in such a position that an opening in its surface facing the observer lies in the continuation of the surface of the perimeter arm. This opening is filled in with diffusing glass bent to take the curvature of the arm and shaded in such a way as to shield the eye of the physician and the observer without changing or interfering with the distribution of light to the perimeter arm. The lamp house rotates with the arm and thus illuminates it uniformly at every point in all meridians.

The principle by which an even illumination of the perimeter arm is secured by this method may be demonstrated as follows. Let S, Fig. 1, be the source of light inlaid in the surface of the arm; P any point in the perimeter arm that is to be illuminated; X the distance from S to P; y the radius of curvature of the arm or the distance of the eye from the arm; a the angle of emission of the light from the source S; and θ the angle of incidence of this light at the point P. Then the intensity at P will be inversely as the square of the distance of P from S or inversely as the square of X, and directly as the cosine of the angle of emission a, $\frac{1}{2}x/y$, and of the angle of incidence θ, also $\frac{1}{2}x/y$. That is,

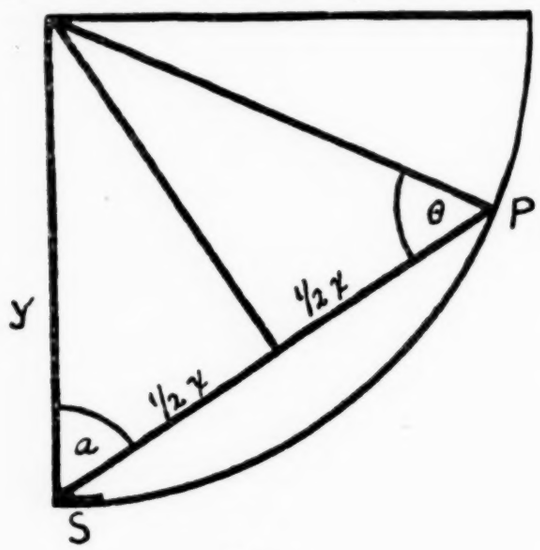


FIG. 1

$$I_1 = I \times \frac{1}{x^2} \times \frac{\frac{1}{2}x}{y} \times \frac{\frac{1}{2}x}{y} = I\,\frac{\frac{1}{4}x^2}{x^2y^2} = \frac{I}{4y^2};$$

in which I_1 is the intensity of light at P and I the intensity at S. From this equation is derived the law of illumination

of the arm,—the intensity of light at any point on the arm is equal to the normal flux of light from the source divided by four times the square of the radius of curvature of the arm.

The method has the following objections. (1) The difficulties in construction are not easy to overcome. (2) Evenness of illumination requires an approximately perfectly diffusing glass. This glass is difficult to obtain and prepare and its transmission is apt to be low. Moreover the light incident on the perimeter arm should approximate daylight in composition. The selective absorption required to correct the light from the lamp to this composition further reduces the intensity enormously. This double loss, first by absorption and second by the somewhat wasteful type of distribution employed, renders it difficult to get an adequate intensity of illumination of the perimeter arm.

Method 2.—When the source of light lies in the perpendicular to the plane of the perimeter arm at its center of curvature, it will be equidistant from every point on the arm; also the angles of emission and incidence of the beam of light will be equal for every point on the arm. A perimeter has been constructed embodying this principle of illumination. This perimeter is shown in Figs. 2–5. Two arcs of the same radius of curvature were constructed at right angles to each other; one a 180° arc, the perimeter arm; the other a 90° arc, the lamp arm, at the end of which is placed the source of light. In order that the source of light shall sustain a fixed relation to the perimeter arm for all positions of that arm, the two arms are fastened together at the center of rotation. About the source is a housing which was designed in such a way as to shield the eye of the patient and the physician without interfering with the distribution of light to the perimeter arm. This housing is made of black japanned iron and is painted a mat black on the inside in order that as nearly as possible all of the light which passes to the perimeter arm shall radiate directly from the lamp filament. Its dimensions are $4\frac{1}{4}$ x $4\frac{1}{4}$ x 5 in. A rectangular aperture $2\frac{1}{8}$ in. wide was cut out of the side of the housing facing the perimeter arm, at the

bottom and for 3 in. back on the two adjacent sides. The relation of the lamp to the aperture of this housing is such that the light radiates freely without shadows from the filament to every point on the perimeter arm. In order that the lamp may be removed when desired, the bottom of the housing is hinged at the back and is held in place by a latch on either

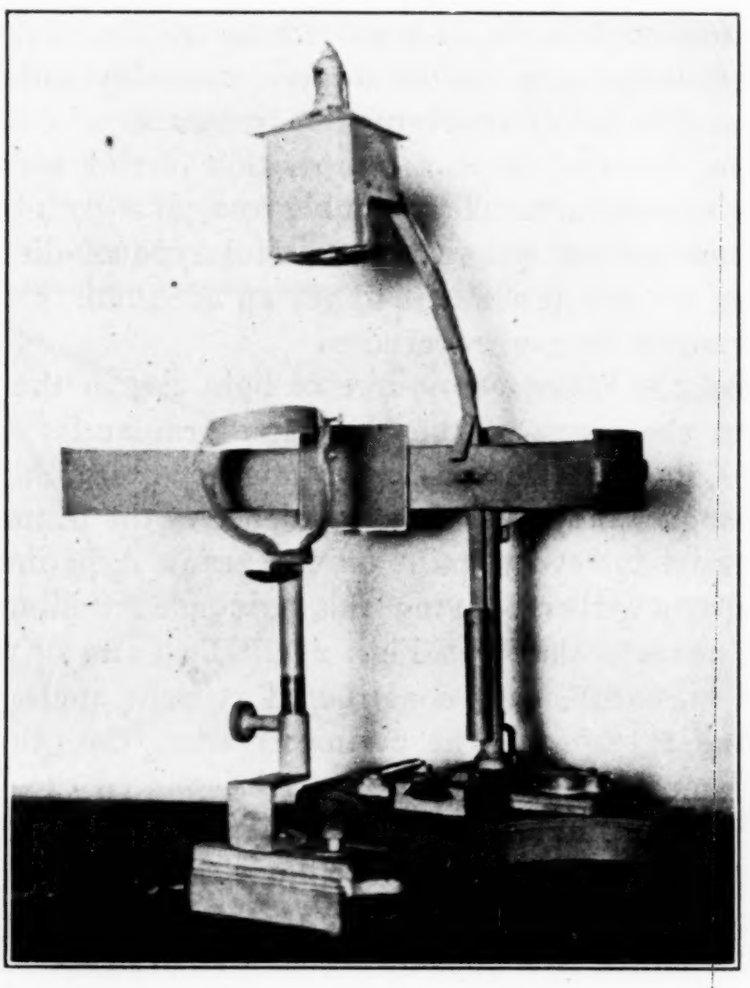


FIG. 2

side. To prevent overheating the housing is well ventilated by especially designed light-tight ventilators, four in the sloping roof of the lamp house and four on each of the sides at the bottom.

Provisions are made in the construction of the lamp house for filtering the light to daylight quality. This is accom-

plished in two ways. (*a*) A well-seasoned 75-watt type C_2 (blue bulb) Mazda lamp operated by ammeter and rheostat control is used as the source of light. This is the ordinary blue bulb commercial lamp and gives only approximate daylight quality. The surface of the bulb is acid-etched to diffuse the light. The intensity of light incident on the perimeter arm when this lamp is used as source of light is 17 foot-candles. (*b*) A type C (clear bulb) Mazda lamp of

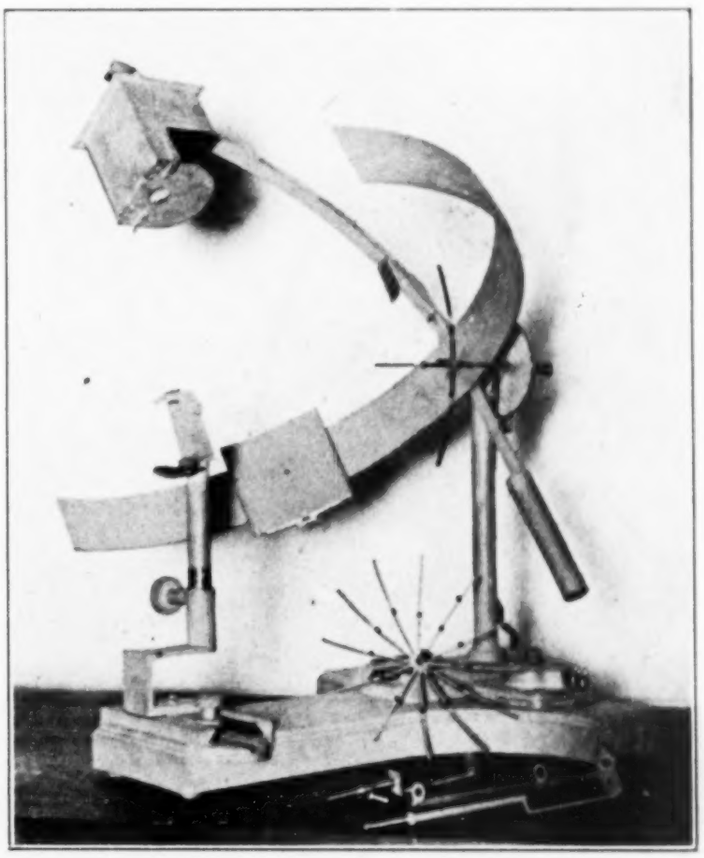


FIG. 3

75, 100 or 150 watts depending upon the intensity of light desired, is used as source. The light from the lamp is filtered to daylight quality by means of a double-etched collar or cylinder of carefully prepared daylight glass 2½ in. broad and 4 in. in diameter, which can be inserted into the bottom of the lamp house so as to fill completely the three-sided aperture. When in position all of the light emitted from this

lamp house must pass through the filter. The collar is securely held in position by three pins so placed in the bottom of the lamp house as to form the apices of an equilateral triangle of the appropriate dimensions. This bottom is hinged so that it can be dropped down for the insertion of the collar and is provided with latches so that it can be held securely in position when raised. The light of a type C Mazda lamp

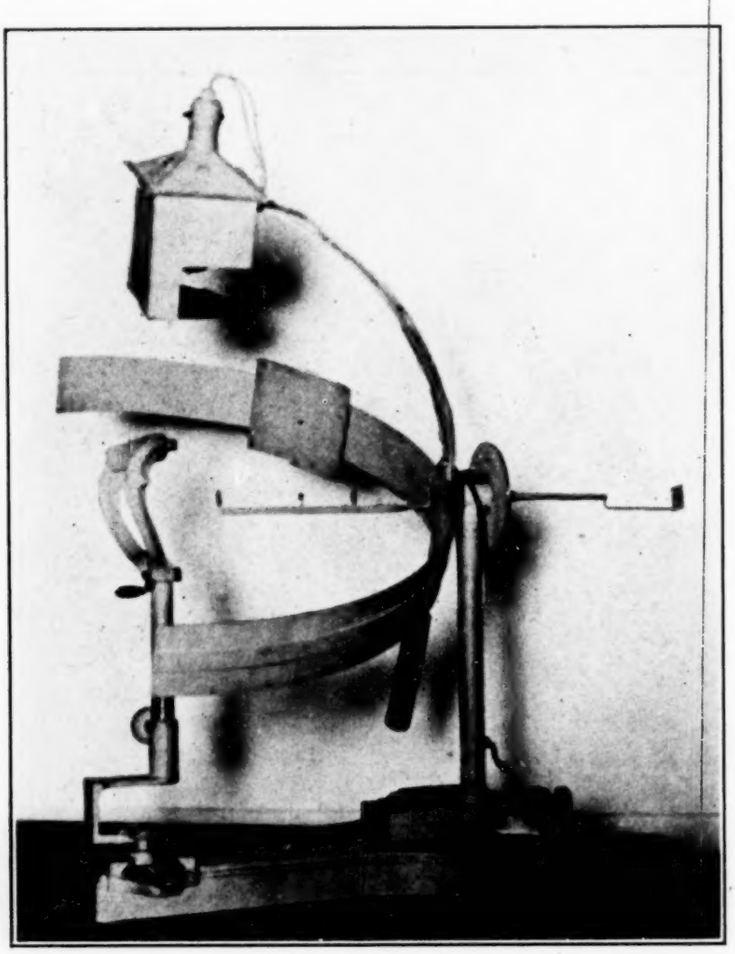


FIG. 4

filtered by this glass is a very close spectro-photometric approximation to north skylight. The coefficient of transmission of the glass is 15 per cent. The light incident through the filter on the perimeter arm from a 100-watt type C Mazda lamp has a value of 2.6 foot-candles,—an intensity very suitable for the taking of a low illumination field the importance

of which in diagnosis will be discussed later. Higher intensity fields may be obtained by using higher wattage lamps.

This perimeter is not difficult to construct or to operate. It provides for a uniform illumination of the perimeter arm in all meridians with light of a good intensity and quality; and with it a precision of control is possible which is com-

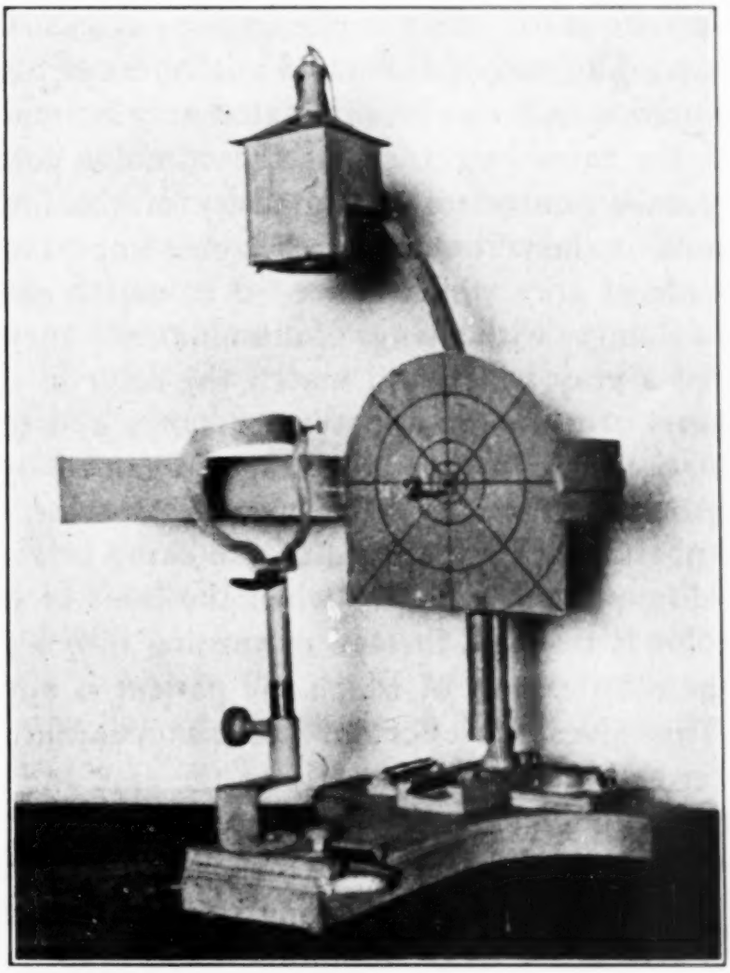


FIG. 5

parable with the work of the physical laboratory. Of the two instruments we have devised, it is without doubt much the more feasible and it is also very probably the more correct in actual practice. Both instruments are correct in theory.

2. *The Brightness of the Preëxposure and the Surrounding Field.*—The brightness of the surface to which the eye is

preëxposed may change the apparent limits in certain meridians as much as 17–20°. A preëxposure lighter than the color gives a dark, and one darker than the color a light after-image. These after-images change profoundly the saturation of color sensation, also its hue. A background or surrounding field lighter or darker than the color produces a similar effect on the limits, but not so great. In this case the disturbing achromatic effect is due to physiological induction or contrast. The variable effect of brightness of preëxposure and surrounding field can be eliminated only by making both a gray of the same brightness as the stimulus color. Here again a precise control of the intensity of the illumination for all points on the perimeter arm becomes important. That is, the shade of gray which is needed to match the color in brightness changes with change of illumination; therefore, the selection of a gray which will match the color in brightness for all points of work presupposes constancy and uniformity of illumination. A further advantage is gained by making the background of the same brightness as the color. That is, when color and background are of the same brightness the stimulus disappears completely when the limit of sensitivity to that color is reached, instead of turning into a gray concerning the colorlessness of which the patient is apt to be in doubt. This gives the effect of the disappearance type of photometer and like it adds greatly to the ease and certainty of making the judgment.

The control of brightness of preëxposure and surrounding field is provided for in the following way. To the stimulus carriage is attached a light aluminum holder, No. 19 B. & S. gauge, grooved to hold a card 5 x 6 in. These cards are covered on one side respectively by grays of the brightness of the four colors, red, yellow, green and blue, of the Hering standard series of pigment papers, as seen in the peripheral retina. At the center of each of these cards is pasted a disc of the appropriate color subtending a visual angle of 1°. To provide for the control of the preëxposure for the stationary method of giving the stimulation, cards identical with the

background cards are provided, covered also on one side with a gray of the brightness of the color. The stimulation by this method is given as follows. The stimulus is placed at the point to be tested and covered with the preëxposure card. The observer is told to take his fixation. At a given signal the stimulus is uncovered for one second and re-covered. In case the moving stimulus method is used, the surrounding field serves as the preëxposure.

The perimeter arm and body are painted a gray of a shade which is approximately mid-gray to the blue and yellow, the darkest and lightest of the stimuli employed. In our own laboratory the perimeter is used on a table painted with the same gray and stands before a gray screen. These latter precautions, however, are not necessary.

When the instrument is supplied to the profession, provision will be made that the stimulus and preëxposure cards, a seasoned lamp, and all other perishable parts can be furnished in the quantities desired.

3. *The Accuracy and Steadiness of Fixation.*—All are familiar with the disturbing effect of inaccuracy and unsteadiness of fixation. If correct and reproducible results are to be obtained, the eye must be accurately placed at the center of the sphere in the surface of which lies the perimeter arm, and the line of sight must not shift from the fixation point while the color observation is being made. As an aid to the correct placement of the eye and a check on its steadiness of fixation, two devices have been provided. (*a*) A small circular mirror is used as a fixation object in which the observer sees the image of his own eye (Fig. 2). When the eye is correctly placed with the line of sight normal to the surface of the mirror at its central point, the fact is indicated to the observer by the position of the image of his pupil and iris as seen in the mirror. Not only is this simple device of service in determining the correct position of the eye, but it aids the observer greatly in holding a steady fixation.

We scarcely need to point out that a fixation object does not afford an accurate control of fixation. Exact checking

methods show that the patient is not always fixating the object when he thinks he is. The only guide to monocular fixation is clearness of seeing and this is a criterion that presents considerable latitude. However with a mirror the patient has an objective check on the position of his eye.

One of the objections to the use of a mirror as a means of controlling fixation is the liability of glare from its surface probably due to a combined specular and diffuse reflection, rendering in proportion as it occurs the clear seeing of the image of the eye by the patient unnecessarily difficult. This objection has been obviated in the instrument described by cutting off the direct radiations from the lamp from the mirror by a narrow shield which can be turned back out of the way when not in use. With the shield in position the eye receives from the mirror only the light which is first reflected from the eye to the mirror and then back to the eye. This device adds greatly to the ease and clearness with which the patient sees his imaged eye. The elimination of the troublesome glare on the surface of the mirror is rendered particularly simple and easy in case of this instrument because of the plan of illumination employed, *i.e.*, the light all comes from a fixed source above and directly in front of the mirror. If the perimeter were illuminated from a window, for example, and its position with reference to the window changed as is needed in order to give as nearly as possible equal illumination of the stimulus in its various positions, the elimination of glare from the surface of the mirror would not be so simple a problem.

Another theoretical objection to the mirror is that the clear seeing of the iris in the mirror requires the eye to focus for twice its distance from the perimeter arm. When this distance is 33 cm., as in the present case, the use of the mirror as a fixation device throws the refracting system of the eye 1.50 diopters out of focus for the colored stimulus on the perimeter arm. Just how important this is when the problem is the mapping of the limits of color sensitivity and not a determination of acuity is difficult to say *a priori*. The poor

imaging for the peripheral field is well known to all who have made determinations of acuity at points far removed from the center of the field. The additional blurring due to an error of focusing of the magnitude described is probably of negligible consequence at points having an excentricity of 30–70 degrees, particularly in determinations of color sensitivity. In any event we have not been able to detect appreciable differences in the limits of sensitivity at 17 foot-candles of illumination with the mirror as fixation device and with the other device provided, in case of which the eye is correctly focused for clear seeing at 33 cm.

(*b*) The second device for the control of fixation is similar in principle to a peep-sight and may be called a parallax or peep-sight device (Fig. 5). A small disc or button (8 mm. in diameter) at the center of rotation of the perimeter arm is viewed through a circular opening (7 mm. in diameter) in a thin round metal disc 9 cm. nearer to the eye. The plane of the disc and the viewing opening are both normal to the line of sight when the eye has its correct position and fixation. When the eye has this position and fixation the relation of size of disc and opening is such that the disc is seen not quite to fill the opening. The disc is painted black, also the edge of the opening, thus when the eye has the proper position and fixation, the edge of the opening is seen concentric to the disc with a narrow ring of the gray of the perimeter arm between. The control afforded by this device is very sensitive. A very slight deviation of the position or fixation of the eye results in the complete or partial extinction of this ring at a point in the direction of the deviation. Like the mirror the device is mounted on a short pin or plug which can be inserted in the fixed tubular axle about which the perimeter arm rotates.

However, while the image in the mirror or the correct sighting of the fixation disc through its opening will indicate to the observer when the line of sight is normal to the surface of the fixation object at its central point, there are two important features in the correct adjustment of the eye over which these devices exercise no control: (*a*) the distance of the

eye from the mirror; and (*b*) the agreement of the meridians of the field of vision as read on the perimeter with the meridians of the retina. In order that it may be known when the eye is at the correct distance from the perimeter arm, a light measuring rod 33 cm. in length is provided, to one end of which is fastened at right angles a small metal disc. In making the adjustment for distance one end of the rod is placed against the mirror at its center and the distance of the perimeter from the observer's eye is changed by means of the coarse screw adjustment to be described later, until the closed lid is just in contact with the metal disc.

Perhaps the simplest device for ensuring a constancy of relation of the meridians of the retina to the meridians of the field of vision as laid off by the perimeter arm, in other words for guarding against a slight tilting of the head to one side or the other, is a mouth bit. We have designed (Fig. 3) a very small and unobjectionable mouth bit of light wood to be changed for each observer, so shaped that it can not be bitten too far forward or back, and thus the distance of the eye from the mirror be changed, or too far to one side or the other. There seems, however, to be an insuperable prejudice against the use of a mouth bit by both the physician and the patient. We have designed, therefore, (Figs. 2, 4 and 5) a head rest which follows approximately the outlines of the forehead, side of the head and face, furnished with a suitably cupped chin rest, the height of which is adjustable. To provide for individual differences in shape and breadth of forehead, an adjustable forehead piece or band of thin spring steel extending well around to the side of the head is screwed at its central point to the forehead piece of the head rest. This forehead band is adjusted to fit foreheads of different shape and breadth by means of a set screw on either side near the two ends of the band. When the chin rest is adjusted to its proper height and the forehead band is made to fit the forehead, the patient's head is held comfortably in position and sufficiently rigid, it is believed, to satisfy the needs of office and clinic work. At least the probability of tilting the head to one side or the other,

thus causing a disagreement of the meridians of the field of vision as indicated by the perimeter readings with the meridians of the retina is very greatly lessened, if not entirely obviated. This it will be remembered, is the especial purpose of the device, the other features of the control being taken care of in other ways.

In order quickly and conveniently to locate the patient's eye at the center of the perimeter system, three adjustments are provided: a rack and pinion to raise and lower the head, a second rack and pinion to shift the head to right or left, and a coarse screw adjustment to change the distance of the perimeter arm from the eye. In the process of getting the eye in position, the patient bites the mouth bit or adjusts the head in the head rest, brings the eye to the level of the fixation device (mirror or peepsight) with the first rack and pinion, and centers its image in the mirror with the second rack and pinion. The physician gets the correct distance of the perimeter arm from the patient's eye by means of the screw adjustment and the measuring rod already referred to. Once these adjustments are made for an eye, they need not be made again during the process of taking the fields for that eye; that is, the biting of the mouth bit or the return of the head to the head rest guarantees that the eye always returns to the same position for which the original adjustments were made.

A very great practical need in a clinic perimeter is a method of controlling fixation for patients who have a central scotoma or pathological blind area. With the eye properly adjusted for taking the fields these patients are not able to see a central fixation object. A device has been constructed (Fig. 3) for controlling the fixation of such patients in the following way. The perimeter arm is made to rotate about a hollow fixed axle. Into this fixed axle telescopes the stem at the end of which is the mirror used for patients with normal or approximately normal central vision, also a hollow stem carrying the device used to control the fixation of patients with a central deficiency. This latter device consists of four light arms or rods

at right angles to each other curved to follow the arc of the perimeter arm, and of sufficient length to provide for all probable breadths of scotoma. Each of these arms carries a small stimulus the distance of which from the center of the field is adjustable. In adjusting the patient's eye by this device the physician looks through a small telescope contained in the hollow tubular axle and lines up the pupil of the patient's eye with the cross hair in the field of the telescope. When the patient's eye is observed to be in position, two or all four of the stimuli as may be desired are adjusted so that they can just be seen by the patient at the edges of the scotoma. These stimuli serve as the control of the patient's fixation, his instructions being so to direct the eye that all are visible.

This fixation device intended to control the fixation during the taking of the fields can be made serviceable for mapping the scotoma itself by adding 12 or more graduated arms equally spaced, making 16 in all, provided with stimuli similar to those already described. Then when the fixation is obtained by the adjustment of the four stimuli designed for that purpose, the further mapping of the scotoma is accomplished by moving the remaining 12 until they are on the edges of the scotoma. Or, if desired, the four stimuli designed for the control of the fixation may not be used for that purpose at all. The physician may watch the patient's eye through the telescope directing the fixation by means of the cross hairs, while all of the stimuli are moved into position on the edges of the blind area. This objective control of the fixation may also be used both in mapping the scotoma or in taking the fields when a central scotoma is present.

If desired the patient's eye can be rendered more visible by reflecting the light from the lamp directly on to it. This is provided for by placing a small oblong mirror of specular metal on the lamp arm at such a position and angle that the light received from the lamp will be reflected on to the iris. This mirror is hinged to the lamp arm and can be turned back against it when not in use. The hinge is provided with a

stop so that when the mirror is turned down it can not go beyond the position required to reflect the light on to the eye. The visibility of the eye in the field of the telescope is considerably increased also by excluding as far as possible scattered light. This was done by putting the objective lens well back in the tube, painting the inside of the tube a mat black and placing on the end of the tube nearer the eye a short tubular hood which shields the opening from the direct rays from the lamp and admits only those reflected from the eye.

Another important need in a clinic perimeter is a method of giving the correct location and fixation to eyes suffering with high myopia. Eyes with myopia ranging from 8–20 diopters would have great difficulty in seeing a fixation object at a distance of 33 cm. Because of the grave pathological changes which take place in the retina and choroid of eyes suffering from high myopia, particularly in the region of the macula and nerve head, it is of great importance to be able to use both the perimeter and the tangent screen to be described later, in the examination of eyes in the more advanced stages of myopia. Three provisions have been made for this. (1) The mirror may be mounted on a rod sufficiently long to permit of its location at any point in the line of sight between the perimeter arm and the eye. This rod may be inserted in the tubular axle on which the perimeter arm rotates. (2) A peep-sight device is provided similar in principle to the one already described and so constructed that it may suffice as a fixation control for values of myopia ranging between 8 and 20 diopters. As shown in Fig. 4 this device has been so contructed that it can be used either for the normal or the myopic eye. That is, it is provided with two sets of fixation targets and viewing openings each at their proper distance from the eye. Both sets are hinged to the narrow steel bar which serves as their support so that either can be turned down out of the way when not in use. The diameter of the target for use with the normal eye is 8 mm.; the diameter of the viewing opening, 7 mm.; and the distance of the opening from the target, 9 cm. The diameter of the

target for use with the myopic eye is 8 mm.; its distance from the perimeter arm is 16 cm. The diameter of the viewing opening is 6 mm. and its distance from the target is 9.5 cm. (3) The perimeter arm may be illuminated with two intensities of light—one carrying the fields well towards the periphery of the retina, the other giving limits narrow enough to fall within the corrected field of the glasses which are worn or may be worn by the patient. This feature provides also for the correction of high astigmatisms, the presence of which make field-taking annoying and uncertain as a diagnostic procedure. There are other advantages too of providing for the taking of fields at more than one intensity of illumination. (*a*) Because of the concentric arrangement of the fibers in the nerve trunk and their order of distribution in the retina, it may be of importance as a point of diagnosis to sample the responses of the retina at different degrees of excentricity. And (*b*) the low illumination fields are in general more sensitive to the influence of the pathological factors which cause the fields to have different breadths. This is largely due to the fact that low illumination fields are narrow fields. That is, sensitivity falls off gradually near the center of the retina, therefore smaller changes of sensitivity are required near the center of the retina to expand or contract the field. It is probably also due in part to the change produced in the state of the retina's sensitivity at low illumination.

The control of fixation for the presbyopic eye also presents a problem to the perimetrist. The eye with a high degree of presbyopia would have considerable difficulty in seeing with the necessary clearness a fixation object at a distance of 33 cm. By the use of the mirror as fixation control this distance is extended to 66 cm. The satisfactory use of the mirror requires that the image of the eye be seen fairly clearly, and an eye without power of accommodation even if there is no hyperopia for far seeing is approximately 1.50 diopters out of focus for an object at a distance of 66 cm. When 1.50 diopters out of focus the eye can not see its image in the mirror with a satisfactory degree of clearness. How-

ever the mirror can be used with a fair degree of satisfaction for lesser degrees of presbyopia.

We have three proposals to make for the control of fixation for the presbyopic eye. (1) The use of the mirror for the lesser degrees of hyperopia. (2) The use of an illumination sufficiently low to bring the color fields within the field of the correcting glasses. And (3) the use of a peep-sight or parallax fixation device (Figs. 3 and 4)[1] similar in principle to the one already described, with the target at a distance great enough to be seen with the necessary degree of clearness by the eye without power of accommodation. Sixty-six cm. has been chosen for this distance because (*a*) the target can be seen at 66 cm. with sufficient clearness to determine whether or not it is at the center of the viewing opening even though the eye is as much as 1.50 diopters out of focus; and (*b*) a greater distance presents difficulty as a matter of feasibility of construction. The device is provided also with a lens the distance of which from the target can be varied from its focal length to that which is needed to render the target clearly visible with 3 diopters or more of accommodation. With this lens at its focal length from the target the rays of light are rendered parallel and the apparent or optical distance of the target is 6 meters. By suitably changing the distance of the lens from the target the apparent distance of the target can be varied over a wide range. By means of this device therefore the apparent distance of the target can be adapted to patients having all possible degrees of presbyopia. However, as already stated, it is quite possible to secure a good control of fixation without the use of the optical attachment.

The fixation device consists of a tube 17.5 cm. long and 8 mm. in diameter, and a light metal rod fastened at one end of the tube and supporting vertically at the other a small plate of hard sheet aluminum 3 x 3 cm., at the center of which is a disc 4 mm. in diameter. This disc and plate serve as the

[1] As already stated the device for control of fixation in high degrees of myopia is also shown in Fig. 3 in position for use. When using either, the other should of course be removed.

fixation target. To use this device as a control of fixation the telescope is removed from the tubular axle of the perimeter and the tube of the fixation device is inserted in the axle to a depth of 4.5 cm. To control the depth of this insertion and thus the distance of the target from the eye, the tube is provided with a stop or shoulder 4.5 cm. from the end inserted. The tubular axle and its extension thus form the opening through which the fixation target is viewed, and when the eye is in position at the center of the perimeter system the target is seen to be centrally located in this opening. The target itself is painted black and the plate at the center of which it is mounted is painted in the same shade of gray as the remainder of the perimeter. The target receives its illumination from the lamp which illuminates the perimeter arm. An unobstructed passage of light from the lamp to the target is possible because of the distance of the target 33 cm. behind the perimeter arm. The lens which is used to vary the optical distance of the target from the eye is mounted in the end of the main tube, facing the target. This tube telescopes to the desired depth into a shorter tube at the end of which on a short supporting rod is the target.

The steadiness of fixation is greatly influenced by the method of giving the stimulation. One of the serious objections to a moving stimulus is the difficulty of holding a steady fixation while the object to be observed is moving. The alternative procedure is the use of a stationary stimulus. That is, the stimulus is placed at the desired point on the perimeter arm and covered with the preëxposure card. The observer takes his fixation and at a given signal the stimulus is exposed and re-covered. By this method of giving the stimulation more time is consumed but a much greater precision of result is possible. A compromise procedure is recommended. That is, the approximate location of the limit is determined with the moving stimulus and the exact location with the stationary stimulus. By this compromise but very little more time is required and there is no sacrifice of precision.

In order to provide for the mapping of the normal blind spot and for the quick detection and mapping of central and paracentral scotomata, it has been deemed advisable to add to the perimeter a tangent screen subtending a visual angle of 60 or more degrees. Provision is made so that this screen can be quickly and conveniently attached to the stimulus carriage and moved into position. The stimulus carriage and the tangent screen have at their exact center a circular opening equal in size to the cross section of the tubular aperture about which the perimeter arm rotates. Thus when the tangent screen is in position, *i.e.*, with its central point in the axis of rotation of the perimeter arm, the tubular opening in the perimeter is continued through to the front surface of the tangent screen. This both provides for the exact adjustment of the screen and permits of the convenient use with it of all of the fixation controls which we have described. Cards of white or black as may be desired with the fields laid off on a tangent scale are provided for the mapping of the area deficient in the light sense, and of grays of the brightness of the colors for mapping the color deficiencies. This is shown in Fig. 5. It is quite possible that the stimulus preëxposure cards used for the field-taking and the larger backgrounds or cards for the tangent screen could be made of thin hard sheet aluminum coated with a flat enamel paint of white, black or grays of the brightness of the colors. If this were done they could be kept clean by washing and the small paper disc of pigment color used as the stimulus for the field taking is all that would have to be renewed as the result of long periods of use. These paints can be bought in a flat black and a flat white. By using the two in the desired proportions the shades of gray desired may be obtained.

In our own work we have found it convenient to use the large screen just described for a quick survey of the field for scotomata and a smaller screen similar to the one used to carry the colored stimulus in field-taking for the actual detailed mapping of the scotomata and the normal blind spot. This screen was made considerably larger than the screens which

serve as backgrounds for the colored stimulus in field-taking in order that it may serve for the mapping of large scotomata and pathologically enlarged blind spots. It is mounted in a carriage of its own and is shaped to take the curvature of the perimeter arm. This screen is intended not as a substitute for the larger screen but as a supplement in cases in which such a supplement is found to be convenient and desirable. This device has the following advantages over the large central tangent screen for the actual mapping of the blind areas. (1) It can be moved to any part of the field from the center out to 90° in any meridian and its center located at the center of the area to be mapped. When the screen is properly centered the mapping can be done as it is on any tangent screen. (2) Upon each screen is drawn 16 meridians, radially from the center of the screen. These meridians are finely graduated so that the limits of sensitivity in any meridian can be read off for the permanent record and transferred to properly planned maps at the convenience of the experimenter. (3) Blind areas are most easily and precisely mapped when the stimulus is made to follow lines radiating from the center of the area. Unless there are such guiding lines it is difficult to pass from within out or from without in consistently when determining the limits of the blind area or when checking up the location of a limit by a second or third determination. On a large fixed screen these lines would have to be drawn specially for each scotoma. (4) All of the evidence points towards the importance of mapping the blind areas with colored stimuli, particularly the Mariotte spot. It is highly important that the determinations for the different colors be made on backgrounds of the same brightness as the colors, as will be shown by blind-spot studies to be published later. It is much more feasible to arrange for this in case of the smaller movable fields than in case of a large fixed field. When the background for the smaller field is laid off in graduated radial lines it can serve for the mapping of many blind spots and scotomata before it need be replaced. In case of a large fixed screen, this superior adaptability and long

service would not be possible. (5) With the movable smaller field it is more nearly possible to map all blind areas under the conditions of equal illumination which obtain in the perimetry of the color fields than it is in the use of the large fixed screen. In the latter case the illumination at which the mapping is done in one part of the field may be quite appreciably different from that at which it is done in another part of the field. The control of illumination, while not so important for mapping the blind areas to the light sense stimuli, is very important in the mapping of the blind areas to color.

With the controls provided in the perimeter recommended, a careful worker can without difficulty reproduce the limits of sensitivity within 1 or 2 degrees.

SOLVING INTERCORRELATIONS BY POLAR COÖRDINATES

BY HERBERT A. TOOPS

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In another place[1] the author has shown how intercorrelations of a number of variables may be efficiently solved on the adding machine by means of transmutation of gross scores into class numbers with the aid of standard grouping tables. The method of solving intercorrelations here presented is efficient in point of speed, although not quite so efficient, and in addition possesses the advantage of yielding a scattered diagram of the data. With reasonable care, computational errors are unlikely to occur. This advantage is secured by the numerous checks obtainable in the plotting method[2] used by the author.

In the first-mentioned method of solving intercorrelations, the transmuted class scores of the Y-variable are borne into the midst of the data sheet by means of carbon copy strips in order that the XY products may be readily computed mentally. This function is performed in the new polar method by a 6-foot radius bar, H, (Fig. 1) which bears the Y-marginal classification successively into the bodies of a number of polar coördinate correlation plots arranged radially about a common center, C. Each of the plots is a small arc-sector of a circle 12 feet in outside diameter. The arrangement of the charts is shown schematically in Fig. 1. This method employs the transmuted class scores derived by the use of grouping tables.[1] The equation employed is:

$$r = \frac{[\Sigma X^2 + \Sigma Y^2 - \Sigma(X - Y)^2]\frac{N}{2} - \Sigma X \cdot \Sigma Y}{\sqrt{N\Sigma X^2 - (\Sigma X)^2}\ \sqrt{N\Sigma Y^2 - (\Sigma Y)^2}}, \qquad (1)$$

[1] Toops, H. A., 'Computing Intercorrelations of Tests on the Adding Machine.' To appear shortly.

[2] Toops, H. A., 'Eliminating the Pitfalls in Solving Correlation; A Printed Correlation Form,' Jour. of Exp. Psych., Vol. 4, No. 6, 1921, pp. 434–447.

in which X and Y are steps of the polar correlation plot, or are transmuted class scores derived from the grouping tables. These transmuted class scores are the classes under which the gross measures would be filed if there were not over eighteen classes covering the total range of variation of the variable.

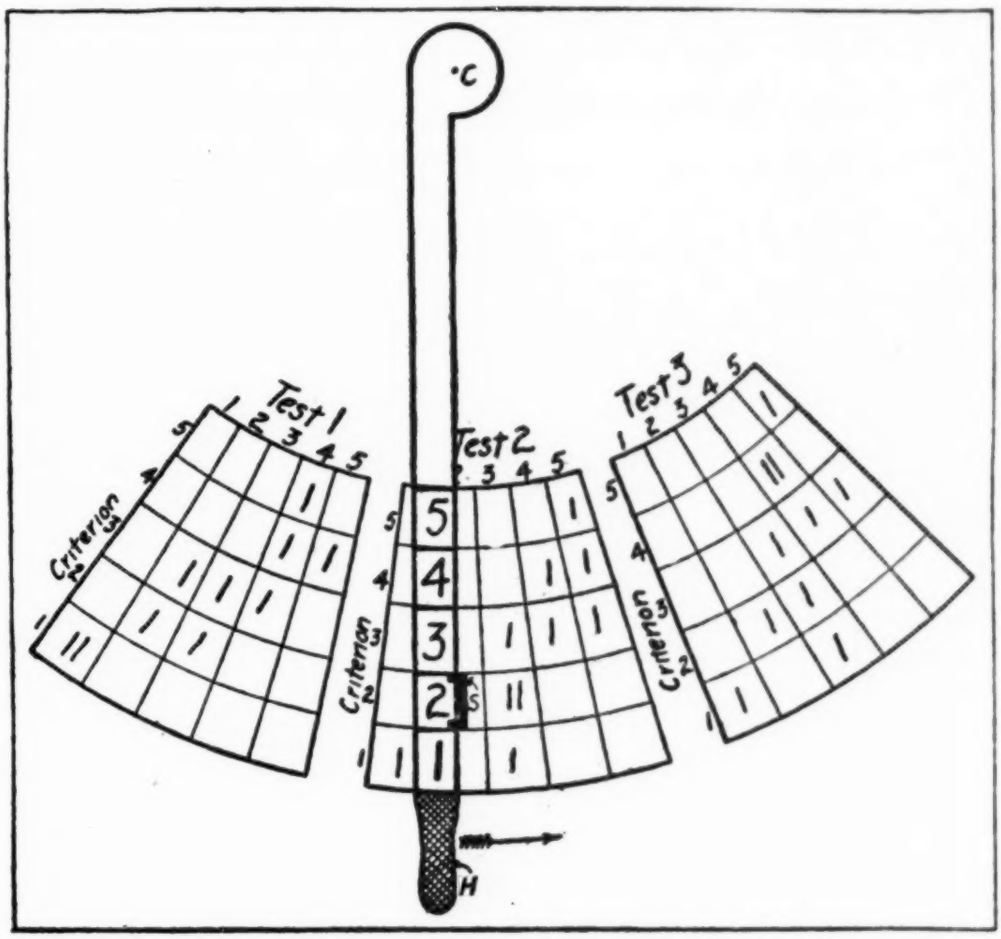

FIG. 1. Schematic diagram of the polar correlation plotting method. This shows how eight to ten polar correlation sheets may be thumbtacked in a semi-circle about a common center, *C*, and all be plotted simultaneously. The slider, *S*, is set at the score common to all charts (criterion score 2, above); thereupon the given subject's scores in the various tests are plotted opposite the slider in the proper columns of each sheet in turn.

It will be seen that (Fig. 2) with a twelve-foot diameter of circle, the small portion of arc required for a scatter diagram of eighteen classes is such as to make the horizontal arcs almost straight lines while the radial lines are almost parallel. The coördinates are thus practically rectangular coördinates.

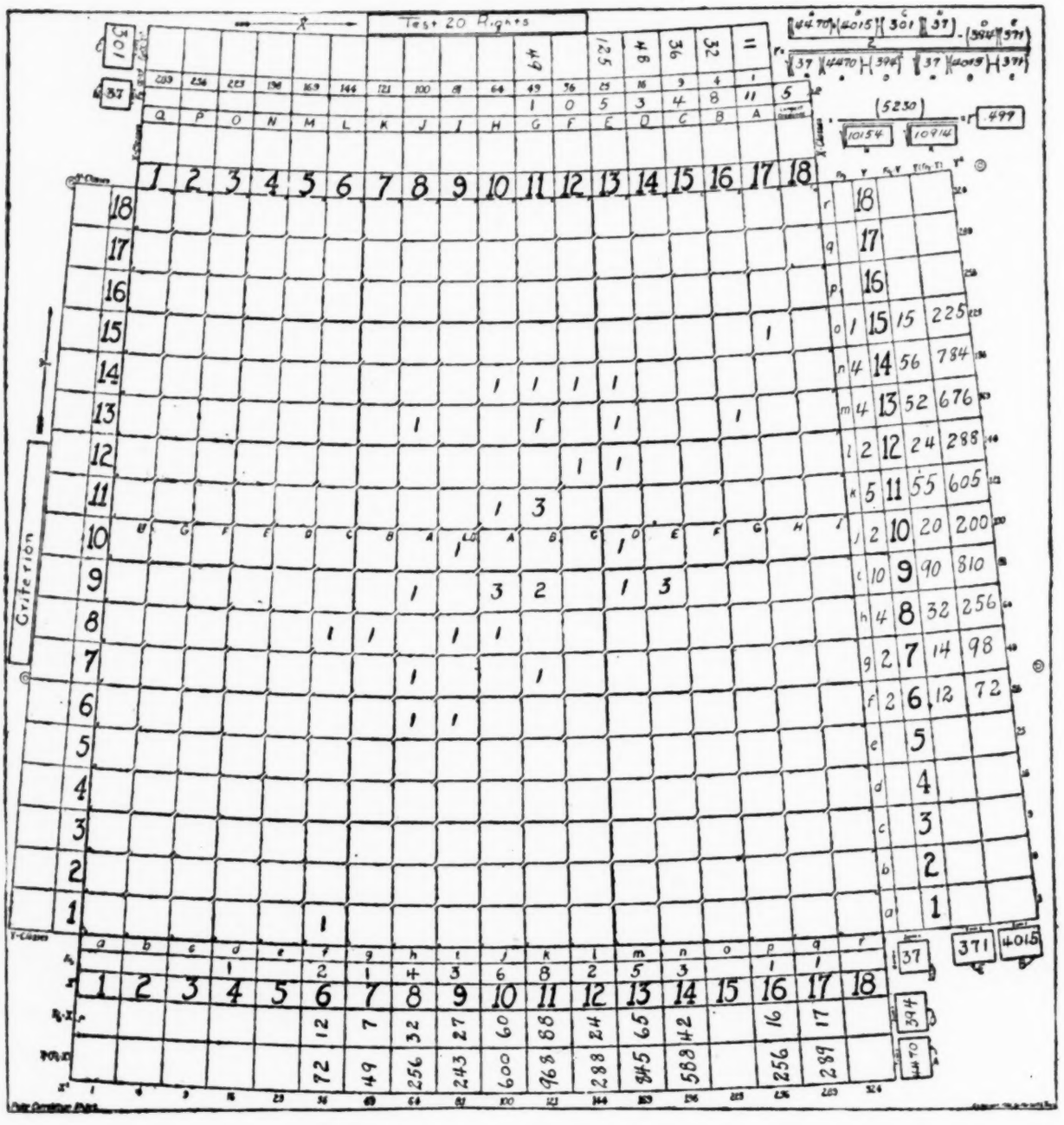

FIG. 2. The computation of r by the polar coördinate sheet method.

In use, a number of the correlation plots are thumbtacked to the top of a soft pine table arranged so that all the radial

lines have a common center.[1] The slider, *S*, on the radial bar is set to indicate the *Y*-class common to the n' correlations being plotted, of Person 1 in the variable, Criterion, say, with which the intercorrelations are at the time being computed. It is evident then that all the check marks for Person 1 on the various tests (plots) will fall on the same horizontal circle indicated by the slider. Experimenter *A* calls the transmuted classes of three tests, Test 1, Test 2, Test 3, to Experimenter *B*, who swings the radius bar to the right to the proper *X*-class successively of the three tests, recording in each a tally mark opposite the slider; *A* then reads the transmuted classes of the next three tests of the same person, and so on until *B* has reached the end of the swing of the bar, which is at the last test of the series of intercorrelations. Then the same procedure is gone through with for Person 2, Person 3, etc., in turn until all *N* cases have thus been plotted. The long bar, if made of thin material, will be found to respond very readily to rapid changes of position. It may readily be lifted entirely from the chart if not pivoted too tightly at the center.

The plotting compartments are sufficiently large, approximately 7/16 inches square at the center of the chart, to prevent plotting errors if reasonable care is taken. The printed classes at both the top and bottom sides of the chart, as well as the heavy vertical lines after every third compartment make errors in the vertical direction easy to avoid if *B* calls back his entries to *A* as rapidly as he writes down the tally marks; errors will never be made in the horizontal direction if the slider is set correctly each time. If errors are made in any plotting, these will be readily detected by comparing the obtained marginal frequency in each case with the standard marginal frequency as soon as a standard (correct) marginal frequency is obtained. (It is well worth the extra time to obtain a correct list of all marginal frequencies by tabulation at the outset.)

[1] The author is revising the chart so as to be used with a revolving table, the bar, *H*, being stationary and charts moving while the plotter remains seated at one point opposite *H*.

The A-diagonal frequencies for this chart are found by adding all frequencies in *both* A-diagonal rows, the eye being guided in this process by the narrow lanes[1] leading from one diagonal compartment to another; the B-diagonal frequencies by adding all frequencies in *both* B-diagonal rows, and so on.

The computation of r follows the procedure outlined in the diagram below (Fig. 3).

Figure 3 is a systematic job analysis solution of the equation,

$$r = \frac{[A + B - C]\frac{N}{2} - D \cdot E}{\sqrt{NA - D^2}\sqrt{NB - E^2}}, \tag{2}$$

arranged so that all similar operations are done at one time. Only the intercorrelations of four variables are shown.

The procedures involved in Fig. 3 are as follows:

1. Enter the ΣX^2-values by variable numbers at the top and bottom of the chart in the designated rows and repeat as ΣY^2 in the left-hand column, by variable numbers, and check. These are systematically added, the ordinate plus abscissa of a given compartment (x, y) in which the solution of r_{xy} is being sought, thus going to make up for the given (x, y) large compartment the $(\Sigma X^2 + \Sigma Y^2)$-values $= A + B$, or the first entry of each large compartment. These sums check for symmetry of rows and columns about the diagonals.

2. Enter the C-values, $\Sigma(X - Y)^2$, as the second entry of each large compartment throughout the table independently from the original data *twice*—once in the upper diagonal half of the table and once in the lower—and then check the entries for symmetry of rows and columns.

3. Subtract C from $A + B$ in all compartments. Check for symmetry of rows and columns about the diagonals to prove the correctness of this operation.

4. Multiply $(A + B - C)$, already obtained in Procedure 3, by $N/2$ if N is an even number; if N is odd, multiply first by N and then enter half of the product (always an integral quantity) as the fourth entry of each large compartment.

[1] The writer is indebted to Dr. A. S. Otis for the suggestion of this helpful device.

Var. No.			I	1	2	3	
Var. No.	$\Sigma X = D$		371	394	61	405	D_y^2
	$\Sigma X^2 = A$		4015	4470	127	4607	
I	371 $\Sigma Y = E$	4015 $\Sigma Y^2 = B$	1.0000	8485 301 8184 151404 146174 5230 .497	4142 2940 1202 22237 22631 − 394 − .121	8622 324 8298 153513 150255 3258 .389	10914
1	394	4470	8485 301 8184 151404 146174 5230 .497	1.0000	4597 3315 1282 23717 24034 − 317 − .101	9077 405 8672 160432 159570 862 .107	10154
2	61	127	4142 2940 1202 22237 22631 − 394 − .121	4597 3315 1282 23717 24034 − 317 − .101	1.0000	4734 3402 1332 24642 24705 − 63 − .025	978
3	405	4607	8622 324 8298 153513 150255 3258 .389	9077 405 8672 160432 159570 862 .107	4734 3402 1332 24642 24705 − 63 − .025	1.0000	6434
ΣX^2 $N \cdot \Sigma X^2$ $(\Sigma X)^2$ $N \cdot \Sigma X^2 - (\Sigma X)^2 = D_x^2$			4015 148555 137641 10914	4470 165390 155236 10154	127 4699 3721 978	4607 170459 164025 6434	

The first entry of a large compartment is $\Sigma X^2 + \Sigma Y^2 = A + B$

" second " " " " " " $\Sigma(X - Y)^2 = C$

" third " " " " " " $A + B - C$

" fourth " " " " " " $N/2(A + B - C)$

" fifth " " " " " " $\Sigma X \cdot \Sigma Y = D \cdot E$

" sixth " " " " " " simplified numerator of r

$$= \frac{N}{2}(A + B - C) - D \cdot E$$

" seventh " " " " " " r (solved by slide rule)

$$= \frac{\text{Numerator}_{xy}}{\sqrt{D_x^2}\sqrt{D_y^2}}$$

FIG. 3. The systematic solution of equation (1) in solving intercorrelations. $N = 37$ persons.

(Note: $A + B - C$ *must always be* an *even* number and $(A + B - C)(N/2)$ an integral number; if this is not the case, C is undoubtedly in error and should be corrected from the original plots. It is assumed that ΣX^2 has been recomputed several times so that there can be no doubt as to the correctness of these values.) These remainders check for symmetry.

5. Enter (ΣX)-values $(= \Sigma Y)$ by variable numbers at left and top of the chart in the proper compartments of the column and row, and check carefully. Multiply these in systematic order, *i.e.*, by rows across the page, *ordinate times abscissa* throughout, to get the $\Sigma X \cdot \Sigma Y$ products for a given compartment, the fifth entry of a large compartment. These products check for symmetry.

6. Subtract the products of Procedure 5 systematically by rows across the page from the result of Procedure 4. These remainders are the simplified numerators of the correlation coefficients. Check for symmetry of rows and columns as a check upon the accuracy of all the preceding work; note carefully whether algebraic signs are correct.

7. Compute the simplified squares of the denominators in systematic fashion at the foot of the table, obtaining $N\Sigma X^2 - (\Sigma X)^2 = D_x^2 = D_y^2$. Enter these values again as D_y^2, in the extreme right-hand column by variable numbers, and check carefully.

8. Solve for r systematically on the slide rule, using the formula

$$r = \frac{\text{Numerator}_{xy}}{\sqrt{D_x^2}\ \sqrt{D_y^2}}, \tag{3}$$

in which D_x^2 and D_y^2 are the simplified denominators respectively of the D_y^2 row and D_x^2 column (result of Procedure 7), denominator coördinates of the xy-compartment, the r of which is being computed. This is to be done at one continuous operation on the slide rule. Using a calculating machine, it is an advantage to compute $1/\sqrt{NA - D^2}$, whereupon the solution for r becomes a multiplication problem, the

numerators of each row being first multiplied by the denominator-reciprocal of the row, and then these products by the denominator reciprocal of the column systematically. Using a calculating machine, some of the above procedures may be dropped out by performing two or more of them at one time. Two people and a calculating machine can solve the equations of several hundred correlation coefficients in a few hours when making use of the above job analysis procedure. The *r*'s finally obtained must check for symmetry of rows and columns in sign, decimal point, and absolute magnitude. To secure maximum speed without confusion, the operations should be done in exactly the order outlined above.

NOTE ON ELECTRIC COUNTERS[1]

BY W. R. MILES

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The automatic counter is a useful accessory in combination with various laboratory apparatus. It is frequently desirable to operate these counters electrically. Several pieces of apparatus recently seen by the writer show that each designer has solved for himself this rather troublesome problem. Probably in many such cases a large saving in time and expense could be made by using the telephone 'p. b. x. message registers,' commercial electric counters which seem to have escaped the notice of most laboratory workers.[2]

The 'message registers' are very well made and are compact. (See Fig. 1.) The counter and actuating electro-

FIG. 1.

magnet are rigidly mounted together in the same frame. The device is arranged for operation in the horizontal position. The pawl connected with the armature and the locking pawl are both kept in position by gravity, but spring tension may easily be made to do this if it is desired to operate the counter in a vertical position. The entire mechanism is rugged and very accessible for adjustment or alteration. The outside dimensions are about 4 cm. wide by 3 cm. high and 15 cm. long. Total weight is less than 0.5 kg. The digits are 5 mm. high, black on white-metal and very legible. There are four counting wheels. These cannot be set back to zero.

The armature is of the hinge type and acts against a spring.

[1] This note has been prepared at the request of scientific colleagues.

[2] Manufactured by Western Electric Co., Inc., Boston, Mass. Prices range from about 5 to 10 dollars, according to production of the different types.

The maximum movement of its free edge is less than 5 mm. The manufacturers state "that these registers may be operated 120 times per minute and the contact must be in a closed position for a quarter of a second and on the open position one quarter of a second." This is a very conservative statement. Using the registers at rated current we have found them to record upwards of 300 contacts in a minute. If the current is increased contacts as short as .04 to .05 sec. can be made to give positive operation. The spring requires nearly .07 sec. to return the armature. Therefore it is possible with this commercial apparatus to register more than 500 contacts per minute for short periods of operation. It is quite impossible to turn up more than one count with one movement of the armature.

There are a variety of message registers from which one may select. Those most likely to be useful are as follows:

Type	Winding	Rated Ohms	Operating	Production
5–B	Single	0.27	1.1 amps.	Special
5–D	Single	34.0	.117 amp.	Special
5–H	Single	0.27	1.4 amps.	Standard
5–L	Inner	37.5	25.5 volts	Standard
	Outer	463.		
5–M	Single	280.0	.036 amp.	Obsolete
5–R	Inner	120.0	.165 amp.	Obsolete
	Outer	600.0		
5–S	Single	5.0	.33 amp.	Standard

Those marked 'obsolete' or 'special' can usually be obtained as there are telephone systems which require them. Most of these mentioned types have a pair of contacts which is closed when the armature is fully moved and a count actually registered. These contacts are serviceable to operate a signal circuit or to provide an auxiliary graphic record which can be inspected for 'chatter,' or failure to operate, if the event counted is quite regular as revolutions of a wheel, number of steps or respirations. We have found this electric counter fast enough for most purposes and a reliable recorder.[1]

[1] See Benedict, Miles, Roth, and Smith, 'Human vitality, etc.,' Carnegie Inst. Washington, Pub. 280, 1919, p. 127.